MATH 16020 PRACTICE EXAM 1

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:________________________________

1. Find the derivative of $S(t) = 4t^3 \tan(t) - \sqrt{t}$

$$\frac{dS}{dt} = \underline{\hspace{6cm}}$$

2. Given the velocity and initial position of a body moving along a coordinate line at time t, find the body's position, $s(t)$, at time t.

$$v(t) = -4t + 2, \qquad s(0) = 3$$

$$s(t) = \underline{\hspace{6cm}}$$

3. Evaluate the definite integral

$$\int_0^{\pi/6} (3\cos(x) - 6)\,dx$$

$$\int_0^{\pi/6} (3\cos(x) - 6)\,dx = \underline{\hspace{6cm}}$$

4. Evaluate the definite integral

$$\int_0^{4} (3e^x + 2)\,dx$$

$$\int_0^{4} (3e^x + 2)\,dx = \underline{\hspace{6cm}}$$

5. A faucet is turned on at 9:00 am and water starts to flow into a tank at the rate of

$$r(t) = 6\sqrt{t}$$

where t is time in hours after 9:00 am and the rate $r(t)$ is in cubic feet per hour.

(a) How much water, in cubic feet, flows into the tank from 10:00 am to 1:00 pm?

Answer:________________________

(b) How many hours after 9:00 am will there be 121 cubic feet of water in the tank?

Answer:________________________

6. During a snowstorm, the rate, in inches per hour, at which the snow falls on a certian town is modeled by the function

$$R'(t) = -\cos(t) - 1.2t + 4$$

where t is measured in hours and $0 \leq t \leq 4$. Based on the model, what is the total amount of snow, in inches, that fell on town from $t = 0$ to $t = 4$? Round to one decimal place.

Answer:______________________________

7. Which derivative rule is undone by integration by substitution?

   (A) Power Rule
   (B) Quotient Rule
   (C) Product Rule
   (D) Chain Rule
   (E) Constant Rule
   (F) None of these

8. Which derivative rule is undone by integration by parts?

   (A) Power Rule
   (B) Quotient Rule
   (C) Product Rule
   (D) Chain Rule
   (E) Constant Rule
   (F) None of these

9. What would be the best substitution to make the solve the given integral?

$$\int e^{2x} \cos(e^{2x}) \sin^3(e^{2x})\, dx$$

$u =$ ______________________________

10. What would be the best substitution to make the solve the given integral?

$$\int \sec^2(5x) e^{\tan(5x)}\, dx$$

$u =$ ______________________________

11. What would be the best substitution to make the solve the given integral?

$$\int \tan(5x)\sec(5x)e^{\sec(5x)}\,dx$$

$u =$ ______________________

12. What would be the best substitution to make the solve the given integral?

$$\int e^y \csc(e^y+1)\cot(e^y+1)\,dy$$

$u =$ ______________________

13. Find the area under the curve $y = 14e^{7x}$ for $0 \leq x \leq 4$.

Area = ______________________

14. Evaluate the definite integral.

$$\int_0^2 (5e^{2x} + 8)\, dx$$

$$\int_0^2 (5e^{2x} + 8)\, dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

15. Evaluate the indefinite integral.

$$\int 18x \cos(5x^2)\, dx$$

$$\int 18x \cos(5x^2)\, dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

16. Evaluate the indefinite integral.

$$\int 9x^3 e^{-x^4}\,dx$$

$$\int 9x^3 e^{-x^4}\,dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

17. Evaluate the indefinite integral

$$\int x(x^2+4)^3\,dx$$

$$\int x(x^2+4)^3\,dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

18. After an oil spill, a company uses oil-eating bacteria to help clean up. It is estimated that t hours after being placed in the spill, the bacteria will eat the oil at a rate of

$$L'(t) = \sqrt{3t+2} \quad \text{gallows per hour.}$$

How many gallons of oil will the bacteria eat in the first 4 hours? Round to 4 decimal places.

Answer:____________________

19. It is estimated that t-days into a semester, the average amount of sleep a college math student gets per day $S(t)$ changes at a rate of

$$\frac{-4t}{e^{t^2}}$$

hours per day. When the semester begins, math students sleep an average of 8.2 hours per day. What is $S(t)$, 2 days into the semester?

Answer:______________________________

20. A biologist determines that, t hours after a bacterial colony was established, the population of bacteria in the colony is changing at a rate given by

$$P'(t) = \frac{5e^t}{1+e^t}$$

million bacteria per hour, $0 \leq t \leq 5$.

If the bacterial colony started with a population of 1 million, how many bacteria, in millions are present in the colony after the 5-hour experiment?

Answer:____________________

21. Evaluate the definite integral.

$$\int_0^{\pi/4} 3\sin(2x)\,dx$$

$\int_0^{\pi/4} 3\sin(2x)\,dx =$ ________________________

22. Evaluate the indefinite integral.

$$\int (x+4)\sqrt{x^2+8x}\,dx$$

$\int (x+4)\sqrt{x^2+8x}\,dx =$ ________________________

23. Evaluate the definite integral.

$$\int_0^9 \frac{dx}{2\sqrt{x}(\sqrt{x}+1)}$$

$\int_0^9 \frac{dx}{2\sqrt{x}(\sqrt{x}+1)}$ ____________________________

24. Evaluate the indefinite integral.

$$\int x\sqrt{x+2}\,dx$$

$$\int x\sqrt{x+2}\,dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

25. A tree is transplanted and after t years is growing at a rate

$$r'(t) = 1 + \frac{1}{(t+1)^2} \qquad \text{meters per year.}$$

After 2 years it has reached a height of 5 meters. How tall was the tree when it was originally transplanted? Round to one decimal place.

Height =____________________

26. The marginal revenue from the sale of x units of a particular product is estimated to be $R'(x) = 50+350xe^{-x^2}$ dollars per unit, and where $R(x)$ is revenue in dollars. What revenue should be expected from the sale of 100 units? Assume that $R(0) = 0$.

$R(100) =$ ________________________

27. Evaluate the indefinite integral

$$\int \frac{\ln(7x)}{x}\,dx$$

$$\int \frac{\ln(7x)}{x}\,dx = \underline{\hspace{6cm}}$$

28. Evaluate

$$\int_1^e \frac{\ln(x^4)}{x}\,dx$$

$$\int_1^e \frac{\ln(x^4)}{x}\,dx = \underline{\hspace{6cm}}$$

29. Evaluate

$$\int_e^4 \frac{dx}{x(\ln(x))^2}$$

$\int_e^4 \frac{dx}{x(\ln(x))^2}$ = ______________________

30. Evaluate the definite integral.

$$\int_0^{\pi/2} (x-1)\sin(x)\,dx$$

$$\int_0^{\pi/2} (x-1)\sin(x)\,dx =$$ ______________________

31. Evaluate

$$\int 3x \ln(x^7)\, dx$$

$$\int 3x \ln(x^7)\, dx = \underline{\hspace{6cm}}$$

32. Evaluate

$$\int x^3 \ln(2x)\, dx$$

$$\int x^3 \ln(2x)\, dx = \underline{\hspace{6cm}}$$

33. Evaluate the indefinite integral

$$\int \sqrt{x}\ln(x)\,dx$$

$$\int \sqrt{x}\ln(x)\,dx = \underline{\hspace{6cm}}$$

34. Evaluate the definite integral.

$$\int_0^3 xe^{3x}\,dx$$

$$\int_0^3 xe^{3x}\,dx = \underline{\hspace{6cm}}$$

35. The population of pink elephants in Dumbo's dreams, in hundreds, t years after the year 1980 is given by

$$P(t) = \frac{e^{5t}}{1 + e^{5t}}$$

What is the average population during the decade between 1980 and 2000?

Answer:______________________________

36. Evaluate the indefinite integral.

$$\int 20x \sin(2x)\, dx$$

$$\int 20x \sin(2x)\, dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

37. The velocity of a cyclist during an hour-long race is given by the function

$$v(t) = 166te^{-2.2t} \text{ mi/hr}, \qquad 0 \le t \le 1$$

Assuming the cyclist starts from rest, what is the distance in miles he traveled during the first hour of the race?

Answer:____________________

38. After t days, the growth of a plant is measured by the function $2000te^{-20t}$ inches per day. What is the change in the height of the plant (in inches) after the first 14 days?

Answer:________________________________