

Please show **all** your work! Answers without supporting work will not be given credit.
Write answers in spaces provided.

Solutions

Name: _____

1. Evaluate the definite integral.

$$\int_0^{\pi/2} (x-1) \sin(x) dx$$

$\frac{u=x-1}{du=dx}$ $\frac{dv=\sin(x) dx}{v=-\cos(x)}$ $uv - \int v du = -(x-1)\cos x \Big|_0^{\pi/2} - \int_0^{\pi/2} (-\cos x) dx$
 $= -(x-1)\cos x \Big|_0^{\pi/2} + \sin(x) \Big|_0^{\pi/2}$
 $= -\left(\frac{\pi}{2}-1\right)\cos\left(\frac{\pi}{2}\right) - [-(0-1)\cos(0)]$
 $+ \sin\left(\frac{\pi}{2}\right) - \sin(0)$
 $= -1 + 1 = 0$

$$\int_0^{\pi/2} (x-1) \sin(x) dx = \boxed{0}$$

2. Evaluate

$$\int 3x \ln(x^7) dx$$

Rewrite $\int 3x(7 \ln(x)) dx = \int 21x \ln x dx$

$$\frac{u=21 \ln(x)}{du=\frac{21}{x} dx}$$

$$\frac{dv=x dx}{v=\frac{x^2}{2}}$$

$$uv - \int v du$$

$$= \frac{21x^2 \ln x}{2} - \int \frac{x^2}{2} \cdot \frac{21}{x} dx$$

$$= \frac{21x^2 \ln x}{2} - \int \frac{21}{2} x dx$$

$$= \frac{21x^2 \ln x}{2} - \frac{21 \cdot x^2}{2 \cdot 2} + C$$

$$= \frac{21x^2 \ln x}{2} - \frac{21x^2}{4} + C$$

$$\int 3x \ln(x^7) dx =$$

$$\frac{21x^2 \ln x}{2} - \frac{21x^2}{4} + C$$

3. Evaluate

$$\int x^3 \ln(2x) dx$$

$$\begin{aligned} \frac{u = \ln(2x)}{du = \frac{1}{2x} \cdot 2 dx} & \quad \frac{dv = x^3 dx}{v = \frac{x^4}{4}} \quad uv - \int v du = \frac{x^4 \ln(2x)}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x} dx \\ & = \frac{x^4 \ln(2x)}{4} - \frac{1}{4} \int x^3 dx \\ & = \frac{x^4 \ln(2x)}{4} - \frac{1}{4} \cdot \frac{x^4}{4} + C \end{aligned}$$

$$\int x^3 \ln(2x) dx = \boxed{\frac{x^4}{4} \ln(2x) - \frac{x^4}{16} + C}$$

4. Evaluate the definite integral.

$$\begin{aligned} \frac{u = x}{du = dx} & \quad \frac{dv = e^{3x} dx}{v = \frac{1}{3} e^{3x}} \quad uv - \int v du \\ & = \frac{x}{3} e^{3x} - \int \frac{1}{3} e^{3x} dx \\ & = \left(\frac{x}{3} e^{3x} - \frac{1}{3} \cdot \frac{e^{3x}}{3} \right) \Big|_0^3 \\ & = \frac{3}{3} e^9 - \frac{1}{9} e^9 - \left[0 - \frac{1}{9} \right] \\ & = \frac{2}{9} e^9 + \frac{1}{9} \end{aligned}$$

$$\int_0^3 5x e^{3x} dx = \boxed{\frac{2}{9} e^9 + \frac{1}{9}}$$

5. Evaluate the indefinite integral.

$$\int_0^{\pi/4} 5x \sin(2x) dx$$

$$\begin{aligned} \underline{u=5x} \quad \underline{dv=\sin(2x) dx} \quad uv - \int v du &= -\frac{5(\pi/4)\cos(2\pi/4)}{2} \\ \underline{du=5 dx} \quad \underline{v=-\frac{\cos(2x)}{2}} &+ \frac{5 \sin(2\pi/4)}{4} \\ &- \left(\frac{5(d)\cos(x)}{2} + \frac{5}{4} \sin(x) \right) \\ &= -\frac{5x\cos(2x)}{2} - \int -\frac{5\cos(2x)}{2} dx \\ &= -\frac{5x\cos(2x)}{2} + \frac{5}{2} \int \cos(2x) dx \\ &= \left(-\frac{5x\cos(2x)}{2} + \frac{5}{2} \frac{\sin(2x)}{2} \right) \Big|_0^{\pi/4} \\ &= 5/4 \end{aligned}$$

$$\int_0^{\pi/4} 5x \sin(2x) dx = \boxed{5/4}$$

$$u-5=2t$$

6. Evaluate the indefinite integral.

$$\begin{aligned} \updownarrow \\ \underline{u=2t+5} \\ \underline{du=2 dt} \\ \underline{\frac{du}{2}=dt} \end{aligned}$$

$$\begin{aligned} \int 4t\sqrt{2t+5} dt &= \int 2 + u^{1/2} \frac{du}{2} = \int 2 + u^{1/2} du = \int (u-5)u^{1/2} du \\ &= \int u^{3/2} - 5u^{1/2} du \\ &= \frac{2}{5} u^{5/2} - 5 \cdot \frac{2}{3} u^{3/2} + C \\ &= \frac{2}{5} (2t+5)^{5/2} - \frac{10}{3} (2t+5)^{3/2} + C \end{aligned}$$

$$\int 4t\sqrt{2t+5} dt = \underline{\hspace{10em}}$$

$$3 \quad \boxed{\frac{2}{5} (2t+5)^{5/2} - \frac{10}{3} (2t+5)^{3/2} + C}$$

7. The velocity of a cyclist during an hour-long race is given by the function

$$v(t) = 166te^{-2.2t} \text{ mi/hr}, \quad 0 \leq t \leq 1$$

Assuming the cyclist starts from rest, what is the distance in miles he traveled during the first hour of the race?

$$\textcircled{1} \int 166te^{-2.2t} dt$$

$$\frac{u=166t}{du=166dt} \quad \frac{dv=e^{-2.2t} dt}{v=\frac{e^{-2.2t}}{-2.2}} \quad uv - \int v du$$

$$= \frac{166t e^{-2.2t}}{-2.2} + \int \frac{e^{-2.2t}}{+2.2} \cdot 166 dt$$

$$= -\frac{166t e^{-2.2t}}{2.2} + \frac{166}{2.2} \cdot \frac{e^{-2.2t}}{-2.2} + C$$

$$= -\frac{166t e^{-2.2t}}{2.2} - \frac{166 e^{-2.2t}}{(2.2)^2} + C$$

$$\textcircled{2} s(0) = 0. \text{ Find } C.$$

$$0 = 0 - \frac{166}{(2.2)^2} + C \rightarrow C = \frac{166}{(2.2)^2}$$

$$\textcircled{3} s(t) = -\frac{166t e^{-2.2t}}{2.2} - \frac{166 e^{-2.2t}}{(2.2)^2} + \frac{166}{(2.2)^2}$$

$$s(1) = -\frac{166}{2.2} e^{-2.2} - \frac{166}{(2.2)^2} e^{-2.2} + \frac{166}{(2.2)^2}$$

$$\approx 22.137$$

Answer: _____

22.137

8. After t days, the growth of a plant is measured by the function $2000te^{-20t}$ inches per day. What is the change in the height of the plant (in inches) after the first 14 days?

$$\int_0^{14} 2000te^{-20t} dt$$

$$\begin{aligned} u &= 2000t & dv &= e^{-20t} dt & uv &- \int v du \\ du &= 2000 dt & v &= \frac{e^{-20t}}{-20} \end{aligned}$$

$$= 2000t \left(\frac{e^{-20t}}{-20} \right) + \int \left(\frac{e^{-20t}}{+20} \right) 2000 dt$$

$$= -100te^{-20t} + 100 \int e^{-20t} dt$$

$$= -100te^{-20t} + 100 \left(\frac{e^{-20t}}{-20} \right)$$

$$= \left(-100te^{-20t} - 5e^{-20t} \right) \Big|_0^{14}$$

$$= 5$$

Answer:

5

9. A model for the ability of a child to memorize information, measured on a scale from 1 to 100, is given by

$$M(t) = 1.9t \ln(t),$$

$2 \leq t \leq 8$, where t is the child's age in years. Find the child's average memorization ability between ages 2 and 7 years. Round to three decimal places.

Average:

$$M_{\text{AVE}}(t) = \frac{1}{7-2} \int_2^7 1.9t \ln(t) dt$$

$$= \int_2^7 \frac{1.9}{5} t \ln(t) dt$$

$$\frac{u = \frac{1.9}{5} \ln(t)}{du = \frac{1.9}{5} \cdot \frac{1}{t} dt} \quad \frac{dv = t dt}{v = \frac{t^2}{2}} \quad uv - \int v du$$

$$= \frac{1.9}{5} \ln(t) \left(\frac{t^2}{2} \right) - \int \frac{1.9}{5} \left(\frac{1}{t} \right) \left(\frac{t^2}{2} \right) dt$$

$$= \frac{1.9}{100} t^2 \ln(t) - \frac{1.9}{100} \int t dt$$

$$= \left[\frac{1.9}{100} t^2 \ln(t) - \frac{1.9}{100} \left(\frac{t^2}{2} \right) \right]_2^7$$

$$\approx 13.315$$

13.315

Answer:

10. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{3x + 1}{x^2(x + 1)^2(x^2 + 1)}$$

(A) $\frac{A}{x^2} + \frac{B}{(x + 1)^2} + \frac{C}{x^2 + 1}$

(B) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x + 1} + \frac{D}{(x + 1)^2} + \frac{E}{x^2 + 1}$

(C) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x + 1} + \frac{D}{(x + 1)^2} + \frac{Ex + F}{x^2 + 1}$

(D) $\frac{A}{x} + \frac{Bx + C}{x^2} + \frac{D}{x + 1} + \frac{Ex + F}{(x + 1)^2} + \frac{Gx + H}{x^2 + 1}$

(E) $\frac{A}{x} + \frac{B}{(x + 1)^2} + \frac{C}{x^2 + 1}$

11. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{7x - 5}{x^2(x^2 + 9)}$$

(A) $\frac{A}{x} + \frac{B}{x} + \frac{Cx + D}{x^2 + 9}$

(B) $\frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + 9}$

(C) $\frac{A}{x} + \frac{Bx + C}{x^2} + \frac{Dx + E}{x^2 + 9}$

(D) $\frac{Ax + B}{x^2} + \frac{Cx + D}{x^2 + 9}$

(E) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x + 3} + \frac{D}{x - 3}$

(F) $\frac{Ax + B}{x^2} + \frac{C}{x + 3} + \frac{D}{x - 3}$

12. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{x^2 + 2x + 3}{(x-1)^2(x-2)(x^2+4)}$$

(A) $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2} + \frac{Dx+E}{x^2+4}$

(B) $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2} + \frac{D}{x^2+4}$

(C) $\frac{A}{x-1} + \frac{Bx+C}{(x-1)^2} + \frac{D}{x-2} + \frac{E}{x^2+4}$

(D) $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2} + \frac{Dx}{x^2+4}$

(E) $\frac{A}{x-1} + \frac{Bx}{(x-1)^2} + \frac{C}{x-2} + \frac{Dx+E}{x^2+4}$

13. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{24}{(x^2-16)^2}$$

(A) $\frac{A}{x-4} + \frac{Bx+C}{(x-4)^2} + \frac{D}{x+4} + \frac{Ex+F}{(x+4)^2}$

(B) $\frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x-2} + \frac{D}{(x-2)^2} + \frac{E}{x+4} + \frac{F}{(x+4)^2}$

(C) $\frac{Ax+B}{(x-4)^2} + \frac{Cx+D}{(x+4)^2}$

(D) $\frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{x+4} + \frac{D}{(x+4)^2}$

(E) $\frac{Ax+B}{x^2-16} + \frac{Cx+D}{(x^2-16)^2}$

(F) $\frac{A}{(x^2-16)^2} + \frac{Bx+C}{(x^2-16)^2}$

Note:
 $(x^2-16)^2$
 $= [(x-4)(x+4)]^2$
 $= (x-4)^2(x+4)^2$

14. Determine the partial fraction decomposition of

$$\frac{7x^2 + 9}{x(x^2 + 3)}$$

$$\begin{aligned} \frac{A}{x} + \frac{Bx + C}{x^2 + 3} &= \frac{A(x^2 + 3) + x(Bx + C)}{x(x^2 + 3)} \\ &= \frac{Ax^2 + 3A + Bx^2 + Cx}{x(x^2 + 3)} \\ &= \frac{(A+B)x^2 + Cx + 3A}{x(x^2 + 3)} \end{aligned}$$

$$(A+B)x^2 + Cx + 3A = 7x^2 + 0x + 9$$

$$\begin{cases} A+B=7 \\ C=0 \\ 3A=9 \rightarrow A=3 \end{cases}$$

So $B=4$

Answer:

$$\boxed{\frac{3}{x} + \frac{4x}{x^2 + 3}}$$

15. Determine the partial fraction decomposition of

$$\frac{4x - 11}{x^2 - 7x + 10}$$

Factor $x^2 - 7x + 10 = (x - 2)(x - 5)$

$$\begin{aligned}\frac{4x - 11}{(x - 2)(x - 5)} &= \frac{A}{x - 2} + \frac{B}{x - 5} \\ &= \frac{A(x - 5) + B(x - 2)}{(x - 2)(x - 5)} \\ &= \frac{(A + B)x + (-5A - 2B)}{(x - 2)(x - 5)}\end{aligned}$$

So $4x - 11 = (A + B)x + (-5A - 2B)$

$$\begin{cases} 4 = A + B & \textcircled{i} \\ -11 = -5A - 2B & \textcircled{ii} \end{cases}$$

Multiply \textcircled{i} by 5 and add $\textcircled{i} + \textcircled{ii}$.

$$\begin{array}{r} 20 = 5A + 5B \\ + \quad -11 = -5A - 2B \\ \hline 9 = 3B \end{array}$$

$$B = 3$$

Plug $B = 3$ into \textcircled{i}

$$4 = A + B$$

$$4 = A + 3$$

$$A = 1$$

Answer:

$$\frac{1}{x - 2} + \frac{3}{x - 5}$$

16. Evaluate $\int \frac{5x^2+9}{x^2(x+3)} dx$

$$\begin{aligned}\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3} &= \frac{Ax(x+3) + B(x+3) + Cx^2}{x^2(x+3)} \\ &= \frac{Ax^2 + 3Ax + Bx + 3B + Cx^2}{x^2(x+3)} \\ &= \frac{(A+C)x^2 + (3A+B)x + 3B}{x^2(x+3)}\end{aligned}$$

$$(A+C)x^2 + (3A+B)x + 3B = 5x^2 + 0x + 9$$

$$\begin{cases} A+C=5 \\ 3A+B=0 \\ 3B=9 \rightarrow B=3 \end{cases}$$

$$\begin{array}{l|l} 3A+B=0 & A+C=5 \\ 3A+3=0 & -1+C=5 \\ 3A=-3 & C=6 \\ A=-1 & \end{array}$$

$$\int -\frac{1}{x} dx + \int \frac{3}{x^2} dx + \int \frac{6}{x+3} dx = -\ln|x| - \frac{3}{x} + 6\ln|x+3| + c$$

$$-\ln|x| - \frac{3}{x} + 6\ln|x+3| + c$$

$$\int \frac{5x^2+9}{x^2(x+3)} dx = \underline{\hspace{10em}}$$

17. Evaluate $\int \frac{x^2 + 2}{x^3 + 3x^2 + 2x} dx$

Factor $x^3 + 3x^2 + 2x = x(x^2 + 3x + 2) = x(x+1)(x+2)$

$$\begin{aligned} \text{So } \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2} &= \frac{A(x+1)(x+2) + Bx(x+2) + Cx(x+1)}{x(x+1)(x+2)} \\ &= \frac{A(x^2 + 3x + 2) + B(x^2 + 2x) + C(x^2 + x)}{x(x+1)(x+2)} \\ &= \frac{(A+B+C)x^2 + (3A+2B+C)x + 2A}{x(x+1)(x+2)} \end{aligned}$$

So $x^2 + 2 = (A+B+C)x^2 + (3A+2B+C)x + 2A$

$1 \cdot x^2 + 0x + 2 = (A+B+C)x^2 + (3A+2B+C)x + 2A$

$$\begin{cases} 1 = A+B+C & \textcircled{i} \\ 0 = 3A+2B+C & \textcircled{ii} \\ 2 = 2A & \textcircled{iii} \end{cases}$$

Solve \textcircled{iii} .

$$\begin{aligned} 2 &= 2A \\ A &= 1 \end{aligned}$$

Plug $A=1$ into \textcircled{i} and \textcircled{ii} .

$$\begin{cases} 1 = 1 + B + C & \textcircled{i} \\ 0 = 3 + 2B + C & \textcircled{ii} \end{cases}$$

Subtract the eqns.

$$\begin{aligned} 1 &= 1 + B + C \\ - (0 &= 3 + 2B + C) \\ \hline 1 &= -2 - B \\ +2 &+2 \\ \hline 3 &= -B \\ B &= -3 \end{aligned}$$

Plug $B=-3$ into \textcircled{i} .

$$\begin{aligned} 1 &= 1 + B + C \\ 1 &= 1 - 3 + C \\ 1 &= -2 + C \\ 3 &= C \end{aligned}$$

Plug $A=1, B=-3, C=3$ into decomposition.

$$\frac{1}{x} + \frac{-3}{x+1} + \frac{3}{x+2}$$

$$\text{So } \int \frac{1}{x} dx + \int \frac{-3}{x+1} dx + \int \frac{3}{x+2} dx$$

$$\int \frac{x^2 + 2}{x^3 + 3x^2 + 2x} dx = \frac{\ln|x| - 3\ln|x+1| + 3\ln|x+2| + C}{}$$

18. Evaluate $\int \frac{9x^2 - 4x + 5}{(x-1)(x^2+1)} dx$

	Bx	C
x	Bx ²	Cx
-1	-Bx	-C

$$\begin{aligned} \text{So } \frac{A}{x-1} + \frac{Bx+C}{x^2+1} &= \frac{A(x^2+1) + (Bx+C)(x-1)}{(x-1)(x^2+1)} \\ &= \frac{Ax^2 + A + Bx^2 - Bx + Cx - C}{(x-1)(x^2+1)} \\ &= \frac{(A+B)x^2 + (C-B)x + (A-C)}{(x-1)(x^2+1)} \end{aligned}$$

$$\text{So } \begin{cases} A+B=9 & \textcircled{i} \\ C-B=-4 & \textcircled{ii} \\ A-C=5 & \textcircled{iii} \end{cases}$$

$$\begin{array}{r} \text{Add } \textcircled{i} \text{ and } \textcircled{ii} \\ A+B = 9 \\ + \quad \cancel{B} + C = -4 \\ \hline A+C = 5 \quad \textcircled{iv} \end{array}$$

$$\begin{array}{r} \text{Add } \textcircled{iii} \text{ and } \textcircled{iv} \\ A-C = 5 \\ + A+C = 5 \\ \hline 2A = 10 \\ A = 5 \end{array}$$

$$\begin{array}{l} \text{Plug } A=5 \text{ into } \textcircled{i} \\ A+B=9 \\ 5+B=9 \\ B=4 \end{array}$$

$$\begin{array}{l} \text{Plug } A=5 \text{ into } \textcircled{iii} \\ A-C=5 \\ 5-C=5 \\ C=0 \end{array}$$

$$\text{So } \frac{5}{x-1} + \frac{4x}{x^2+1}$$

$$\int \frac{5}{x-1} dx + \int \frac{4x}{x^2+1} dx$$

$u = x^2+1$
 $du = 2x dx$

$$= 5 \ln|x-1| + 2 \ln|x^2+1| + C$$

$$\int \frac{x^2+2}{x^3+3x^2+2x} dx =$$

$$\frac{5 \ln|x-1| + 2 \ln|x^2+1| + C}{}$$

19. Evaluate $\int \frac{3x^2 + 3x + 15}{x^3 + 5x^2} dx$

① Factor denominator

$$x^3 + 5x^2 = x^2(x+5)$$

② Decomposition

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+5}$$

③ Combine fractions

$$\frac{Ax(x+5) + B(x+5) + Cx^2}{x^2(x+5)}$$

$$= \frac{(A+C)x^2 + (5A+B)x + (5B)}{x^2(x+5)}$$

④ Old numerator = New numerator

$$3x^2 + 3x + 15 = (A+C)x^2 + (5A+B)x + 5B$$

⑤ System of eqn based on ④

$$\begin{cases} 3 = A+C & \text{(i)} \\ 3 = 5A+B & \text{(ii)} \\ 15 = 5B & \text{(iii)} \end{cases}$$

Solve (iii)

$$15 = 5B \Rightarrow B = 3$$

Plug $B = 3$ into (ii)

$$3 = 5A + 3$$

$$0 = 5A$$

$$A = 0$$

Plug $A = 0$ into (i)

$$3 = 0 + C$$

$$C = 3$$

⑥ Plug $A = 0, B = 3, C = 3$ into ②

$$\frac{3}{x^2} + \frac{3}{x+5}$$

$$\Rightarrow \int \frac{3}{x^2} + \frac{3}{x+5} dx$$

$$= \int 3x^{-2} + \frac{3}{x+5} dx$$

$$= -3x^{-1} + 3\ln|x+5| + C$$

$$= -\frac{3}{x} + 3\ln|x+5| + C$$

$$\int \frac{3x^2 + 3x + 15}{x^3 + 5x^2} dx = \boxed{-\frac{3}{x} + 3\ln|x+5| + C}$$

20. Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \frac{\sin x}{1 - \cos x} dx$$

- (A) It is improper because of a discontinuity at $x = \pi/6$
- (B) It is improper because of a discontinuity at $x = \pi/4$
- (C) It is improper because of a discontinuity at $x = \pi/3$
- (D) It is improper because of a discontinuity at $x = 0$
- (E) It is improper because of a discontinuity at $x = \pi/2$
- (F) It is proper since it is defined on the interval $[0, \pi/2]$.

$$1 - \cos x = 0$$
$$1 = \cos x$$
$$x = 0, \pi, 2\pi$$

21. Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \tan(x) dx$$

- (A) It is improper because of a discontinuity at $x = \pi/6$
- (B) It is improper because of a discontinuity at $x = \pi/4$
- (C) It is improper because of a discontinuity at $x = \pi/3$
- (D) It is improper because of a discontinuity at $x = 0$
- (E) It is improper because of a discontinuity at $x = \pi/2$
- (F) It is proper since it is defined on the interval $[0, \pi/2]$.

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$$

22. Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \cos(x) dx$$

- (A) It is improper because of a discontinuity at $x = \pi/6$
- (B) It is improper because of a discontinuity at $x = \pi/4$
- (C) It is improper because of a discontinuity at $x = \pi/3$
- (D) It is improper because of a discontinuity at $x = 0$
- (E) It is improper because of a discontinuity at $x = \pi/2$
- (F) It is proper since it is defined on the interval $[0, \pi/2]$.

$\rightarrow \cos(x)$ is defined everywhere.

Bonus do this question w/ all trig functions

23. Which of the following integrals are diverges?

I. $\int_1^{\infty} \frac{5}{\sqrt{x}} dx$

II. $\int_1^{\infty} \frac{3}{x^2} dx$

III. $\int_1^{\infty} \frac{10}{x} dx$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I and III only

(F) I, II, and III

Ⓘ $\int_1^{\infty} 5x^{-1/2} dx = 5 \cdot \frac{2}{1} x^{1/2} \Big|_1^{\infty} = \infty$
 \Rightarrow diverges

Ⓡ $\int_1^{\infty} 3x^{-2} dx = -3x^{-1} \Big|_1^{\infty} = -\frac{3}{x} \Big|_1^{\infty} = 0 + 3$

Ⓢ $\int_1^{\infty} \frac{10}{x} dx = 10 \ln|x| \Big|_1^{\infty} = \infty$
 \Rightarrow diverges

24. Which of the following integrals are improper?

I. $\int_0^{\pi/4} \cos(x) dx$

II. $\int_0^{\pi/4} \tan(2x) dx$

III. $\int_{\pi/4}^{\pi/2} \csc(x) dx$

IV. $\int_{\pi/4}^{\pi/2} \sec\left(\frac{x}{2}\right) dx$

(A) II and IV only

(B) I and II only

(C) I and IV only

(D) I and III only

(E) II, III and IV only

(F) II only

Ⓘ $\cos(x)$ is defined everywhere \Rightarrow proper

Ⓡ $\tan(2x) = \frac{\sin(2x)}{\cos(2x)} \Rightarrow \cos(2x) = 0$ when $2x = \frac{\pi}{2}$
 $x = \frac{\pi}{4}$

and $\frac{\pi}{4}$ is a bound \Rightarrow improper

Ⓢ $\csc(x) = \frac{1}{\sin(x)} \Rightarrow \sin(x) = 0$ when $x = 0, \pi, 2\pi, \dots$
 which are none of the bounds \Rightarrow proper

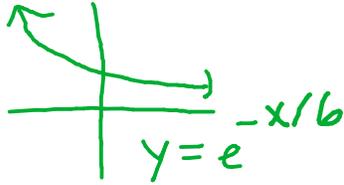
Ⓢ $\sec\left(\frac{x}{2}\right) = \frac{1}{\cos\left(\frac{x}{2}\right)} \Rightarrow \cos\left(\frac{x}{2}\right) = 0$ when $\frac{x}{2} = \frac{\pi}{2}$
 $x = \frac{\pi}{4}$

and $\frac{\pi}{4}$ is a bound \Rightarrow improper

25. Evaluate the following integral;

$$\int_0^{\infty} e^{-x/6} dx$$

$$\begin{aligned} &= \lim_{N \rightarrow \infty} \int_0^N e^{-x/6} dx = \lim_{N \rightarrow \infty} \left(-6e^{-x/6} \right) \Big|_0^N \\ &= \lim_{N \rightarrow \infty} \left(-6e^{-N/6} + 6 \right) = 6 \end{aligned}$$

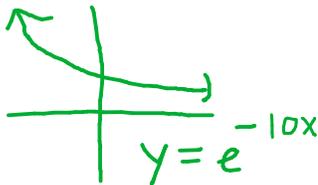


$$\int_0^{\infty} e^{-x/6} dx = \boxed{6}$$

26. Evaluate the following integral;

$$\int_0^{\infty} \frac{7}{e^{10x}} dx$$

$$\begin{aligned} \int_0^{\infty} 7e^{-10x} dx &= \lim_{N \rightarrow \infty} \int_0^N 7e^{-10x} dx = \lim_{N \rightarrow \infty} \left(\frac{7e^{-10x}}{-10} \right) \Big|_0^N \\ &= \lim_{N \rightarrow \infty} \left(\frac{7e^{-10N}}{-10} + \frac{7}{10} \right) = 0 + \frac{7}{10} \end{aligned}$$

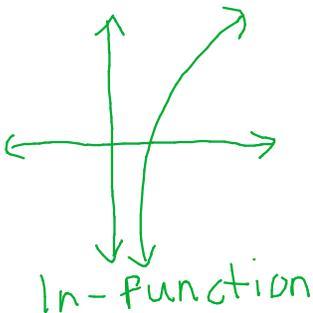


$$\int_0^{\infty} \frac{7}{e^{10x}} dx = \boxed{7/10}$$

27. Evaluate the definite integral

$$\int_2^{\infty} \frac{dx}{5x+2}$$

$$\begin{aligned} \lim_{N \rightarrow \infty} \int_2^N \frac{dx}{5x+2} & \quad \begin{array}{l} u=5x+2 \\ du=5dx \\ \frac{du}{5}=dx \end{array} \\ &= \lim_{N \rightarrow \infty} \int \frac{1}{5} \frac{1}{u} du = \lim_{N \rightarrow \infty} \frac{1}{5} \ln|u| = \lim_{N \rightarrow \infty} \frac{1}{5} \ln|5x+2| \Big|_2^N \\ &= \lim_{N \rightarrow \infty} \left(\frac{1}{5} \ln|5N+2| - \frac{1}{5} \ln|2| \right) = \infty \end{aligned}$$



$$\int_2^{\infty} \frac{dx}{5x+2} = \boxed{\infty}$$

28. Evaluate the definite integral

$$\int_4^{13} \frac{dx}{\sqrt{x-4}}$$

$$\begin{aligned}
 &= \int_4^{13} (x-4)^{-1/2} dx \\
 &= 2(x-4)^{1/2} \Big|_4^{13} \\
 &= 2(13-4)^{1/2} - 2(4-4)^{1/2} \\
 &= 6
 \end{aligned}$$

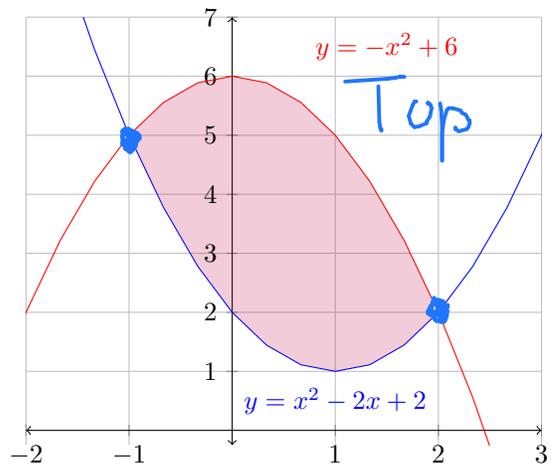
$$\int_4^{13} \frac{dx}{\sqrt{x-4}} = \boxed{6}$$

29. Set up the integral that computes the **AREA** shown to the right with respect to x .

DON'T COMPUTE IT!!!

$$\int_{-1}^2 (-x^2 + 6) - (x^2 - 2x + 2) dx$$

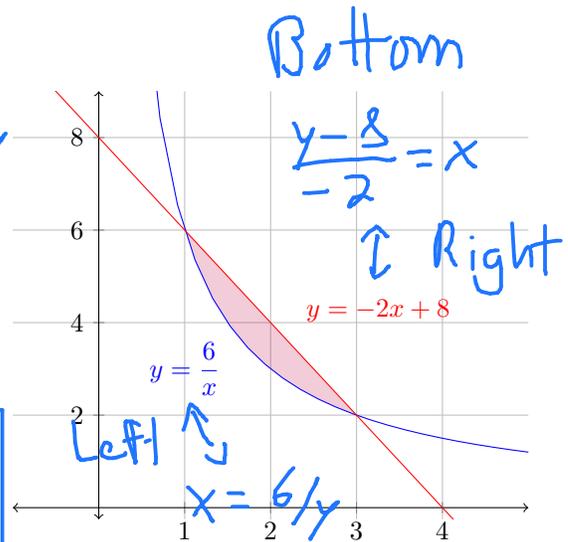
Area = _____



30. Set up the integral that computes the **AREA** shown to the right with respect to y .

DON'T COMPUTE IT!!!

$$\text{Area} = \int_2^6 \left(\frac{y-8}{-2} \right) - \frac{6}{y} dy$$



31. Set up the integral that computes the **AREA** with respect to x of the region bounded by

$$y = \frac{2}{x} \text{ and } y = -x + 3$$

→ dx problem

Bounds:

$$\frac{2}{x} = -x + 3$$

$$2 = -x^2 + 3x$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

$$x = 1, 2$$

Test Pt: $x = 1.5$

$$y = \frac{2}{x} \Rightarrow y = \frac{2}{1.5} = \frac{4}{3} \approx 1.33 \rightarrow \text{Bottom}$$

$$y = -x + 3 \Rightarrow y = -1.5 + 3 = 1.5 \rightarrow \text{Top}$$

$$\text{Area} = \int_1^2 \left(-x + 3 - \frac{2}{x} \right) dx$$

32. Set up the integral that computes the **AREA** with respect to x of the region bounded by

$$y = x \text{ and } y = 7x - x^2$$

Bounds: $x = 7x - x^2$

$$0 = 6x - x^2$$

$$0 = x(6-x)$$

$$x = 0, 6$$

Test Pt: $x = 1$

$$y = x \xrightarrow{x=1} y = 1 \rightarrow \text{Small}$$

$$y = 7x - x^2 \xrightarrow{x=1} y = 6 \rightarrow \text{Big}$$

$$= \int_0^6 (6x - x^2) dx$$

$$= \left(3x^2 - \frac{x^3}{3} \right) \Big|_0^6$$

$$= 36$$

$$\text{Area} = \boxed{36}$$

$$A = \int_0^6 (7x - x^2) - x \, dx$$

33. Find the area of the region bounded by $y = 6x - x^2$ and $y = 2x^2$.

Bounds:
 $6x - x^2 = 2x^2$
 $6x - 3x^2 = 0$
 $3x(2 - x) = 0$
 $x = 0, 2$

Test Pt: $x = 1$
 $y = 6x - x^2 \Rightarrow y = 5 \rightarrow \text{Top}$
 $y = 2x^2 \Rightarrow y = 2 \rightarrow \text{Bottom}$

$$A = \int_0^2 [(6x - x^2) - 2x^2] dx$$

$$= \int_0^2 (6x - 3x^2) dx$$

$$= (3x^2 - x^3) \Big|_0^2 = 4$$

Area = _____



34. Find the area bounded by the following curves.

$$x = y^2 + 24 \text{ and } x = 10y$$

Bounds: $y^2 + 24 = 10y$
 $y^2 - 10y + 24 = 0$
 $(y - 6)(y - 4) = 0$
 $y = 4, 6$

Test Pt: $y = 5$

$x = y^2 + 24 \xrightarrow{y=5} x = 49 \rightarrow \text{Small}$

$x = 10y \xrightarrow{y=5} x = 50 \rightarrow \text{Big}$

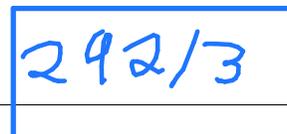
$$A = \int_4^6 10y - (y^2 - 24) dy$$

$$= \int_4^6 10y - y^2 + 24 dy$$

$$= \left(5y^2 - \frac{y^3}{3} + 24y \right) \Big|_4^6$$

$$= \frac{292}{3}$$

Area = _____



35. Find the area of the region bounded by $y = 2x - x^2$ and $y = x^2$.

Bounds:

$$2x - x^2 = x^2$$

$$2x - 2x^2 = 0$$

$$2x(x-1) = 0$$

$$x = 0, 1$$

$$A = \int_0^1 (2x - x^2) - x^2 dx$$

$$= \int_0^1 2x - 2x^2 dx$$

$$= \left(\frac{2x^2}{2} - \frac{2x^3}{3} \right) \Big|_0^1 = \frac{1}{3}$$

Test Pt: $x = \frac{1}{2}$

$$y = 2x - x^2 \rightarrow y\left(\frac{1}{2}\right) = \frac{3}{4} \rightarrow \text{Top}$$

$$y = x^2 \rightarrow y\left(\frac{1}{2}\right) = \frac{1}{4} \rightarrow \text{Bottom}$$

Area =

1/3

36. Calculate the **AREA** of the region bounded by the following curves.

$$x = 100 - y^2 \text{ and } x = 2y^2 - 8$$

Bounds:

$$100 - y^2 = 2y^2 - 8$$

$$108 = 3y^2$$

$$36 = y^2$$

$$y = \pm 6$$

$$A = \int_{-6}^6 (100 - y^2) - (2y^2 - 8) dy$$

$$= \int_{-6}^6 (108 - 3y^2) dy$$

$$= (108y - y^3) \Big|_{-6}^6$$

$$= 864$$

Test Pt: $y = 0$

$$x = 100 - y^2 \rightarrow x = 100 \rightarrow \text{Right}$$

$$x = 2y^2 - 8 \rightarrow x = -8 \rightarrow \text{Left}$$

Area =

864

37. Calculate the **AREA** of the region bounded by the following curves.

$$y = x^3 \quad \text{and} \quad y = x^2$$

Bounds:

$$x^3 = x^2$$

$$x^3 - x^2 = 0$$

$$x^2(x-1) = 0$$

$$x \geq 0, 1$$

Test Pt: $x = \frac{1}{2}$

$$y = x^3 \rightarrow y = \frac{1}{8} \rightarrow \text{Bottom}$$

$$y = x^2 \rightarrow y = \frac{1}{4} \rightarrow \text{Top}$$

$$\begin{aligned} A &= \int_0^1 (x^2 - x^3) dx \\ &= \left(\frac{x^3}{3} - \frac{x^4}{4} \right) \Big|_0^1 \\ &= \frac{1}{3} - \frac{1}{4} = \frac{1}{12} \end{aligned}$$

Area =

1/12

38. After t hours studying, one student is working $Q_1(t) = 25 + 9t - t^2$ problems per hour, and a second student is working on $Q_2(t) = 5 - t + t^2$ problems per hour. How many more problems will the first student have done than the second student after 10 hours?

$$\begin{aligned} &\int_0^{10} Q_1(t) - Q_2(t) dt \\ &= \int_0^{10} (25 + 9t - t^2) - (5 - t + t^2) dt \\ &= \int_0^{10} (20 + 10t - 2t^2) dt \\ &= \left(20t + 5t^2 - \frac{2}{3}t^3 \right) \Big|_0^{10} \\ &= \frac{100}{3} \end{aligned}$$

Answer:

100/3

39. The birthrate of a particular population is modeled by $B(t) = 1000e^{0.036t}$ people per year, and the death rate is modeled by $D(t) = 725e^{0.019t}$ people per year. How much will the population increase in the span of 10 years? ($0 \leq t \leq 20$) Round to the nearest whole number.

$$\int_0^{10} B(t) - D(t) dt = \int_0^{10} 1000e^{0.036t} - 725e^{0.019t} dt$$
$$= \left(\frac{1000}{0.036} e^{0.036t} - \frac{725}{0.019} e^{0.019t} \right) \Big|_0^{10}$$

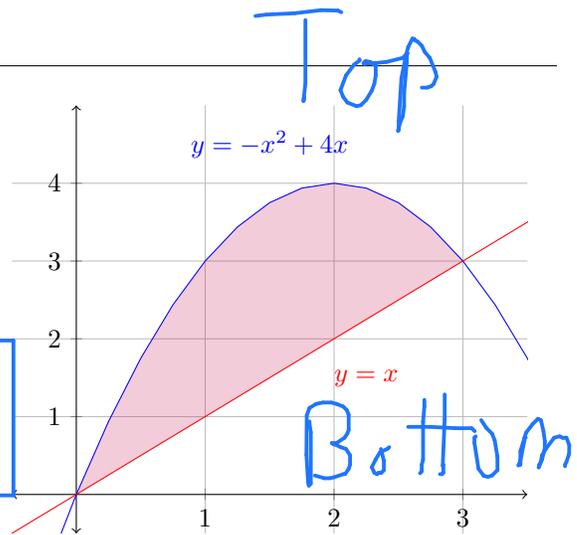
$$\approx 4052$$

Answer: _____

4052

40. Let R be the region shown below. Set up the integral that computes the **VOLUME** as R is rotated around the **x-axis**.

DON'T COMPUTE IT!!!



$$\pi \int_0^3 [(-x^2 + 4x)^2 - (x)^2] dx$$

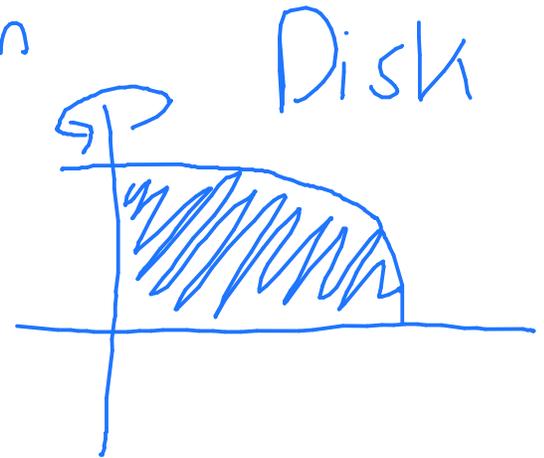
Volume = _____

41. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = \sqrt{16-x}, \quad y = 0 \quad \text{and} \quad x = 0$$

about the **y-axis** \Rightarrow dy problem

$$\begin{aligned} y &= \sqrt{16-x} \\ y^2 &= 16-x \\ x &= 16-y^2 \end{aligned}$$



Bounds: Given $y=0$

Plug $x=0$ into $y = \sqrt{16-x}$

$$y = \sqrt{16-x}$$

$$y = \sqrt{16}$$

$$y = 4$$

$$\pi \int_0^4 (16-y^2)^2 dy$$

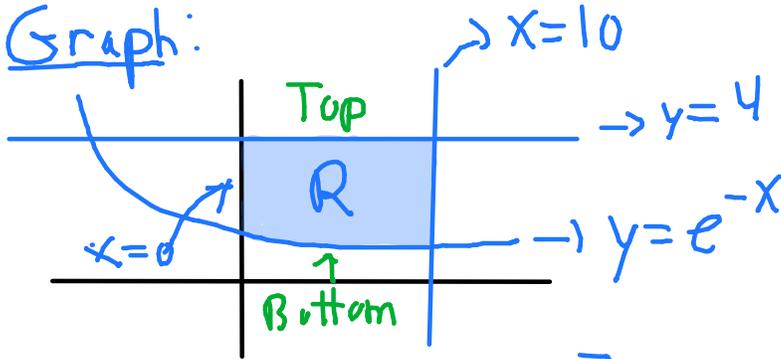
Volume = _____

42. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = e^{-x}, y = 4 \quad x = 0 \quad \text{and} \quad x = 10$$

about the x -axis

$\circ \Rightarrow dx$



$$V = \pi \int_0^{10} [4^2 - (e^{-x})^2] dx$$

$$\pi \int_0^{10} (16 - e^{-2x}) dx$$

Volume = _____

43. Find the volume of the solid that results by revolving the region enclosed by the curves $y = \frac{5}{x}$, $y = 0$, $x = 5$, and $x = 7$ about the x -axis

$\circ \Rightarrow dx$



$$\begin{aligned} V &= \pi \int_5^7 \left(\frac{5}{x}\right)^2 dx \\ &= \pi \int_5^7 \frac{25}{x^2} dx \\ &= 25\pi \int_5^7 x^{-2} dx \\ &= 25\pi \left(-\frac{1}{x}\right) \Big|_5^7 \end{aligned}$$

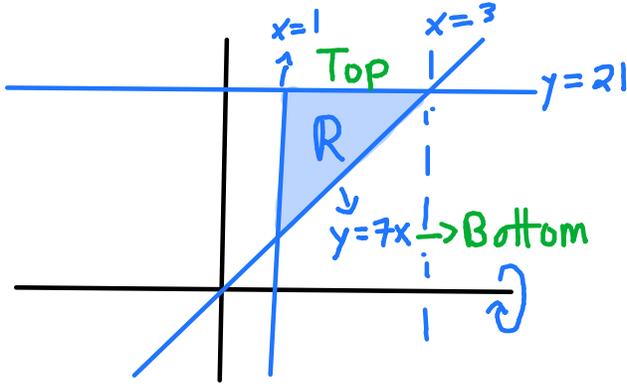
$$= \frac{10\pi}{7}$$

Volume = _____

44. Find the **VOLUME** of the region bounded by

$$y = 7x, \quad y = 21 \quad x = 1 \quad \text{and} \quad x = 3$$

around the x -axis $\rightarrow dx$



Washer

$$\begin{aligned} V &= \pi \int_1^3 [21^2 - (7x)^2] dx \\ &= \pi \int_1^3 (441 - 49x^2) dx \\ &= \pi \left(441x - \frac{49x^3}{3} \right) \Big|_1^3 \\ &= \frac{1274}{3} \pi \end{aligned}$$

Volume =

$$\boxed{\frac{1274\pi}{3}}$$

45. Find the **VOLUME** of the region bounded by

$$y = 7x, \quad y = 0 \quad x = 1 \quad \text{and} \quad x = 3$$

around the x -axis $\rightarrow dx$



Disk

$$\begin{aligned} V &= \pi \int_1^3 (7x)^2 dx \\ &= \pi \int_1^3 49x^2 dx \\ &= \pi \left(\frac{49x^3}{3} \right) \Big|_1^3 \\ &= \frac{49\pi}{3} (3^3 - 1) \end{aligned}$$

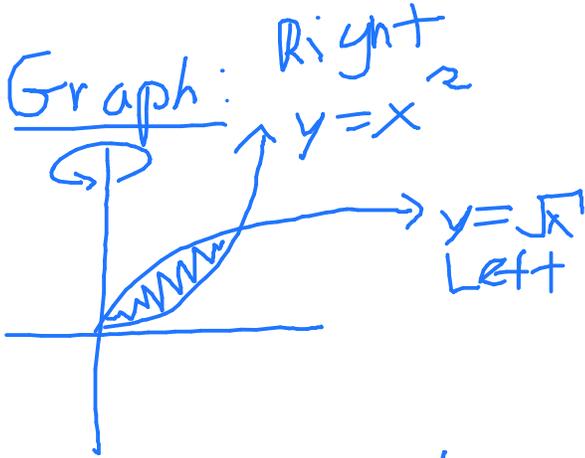
Volume =

$$= \boxed{\frac{1274}{3} \pi}$$

46. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = x^2, \text{ and } y = \sqrt{x}$$

about the y-axis



Bounds:

$$\sqrt{y} = y^2$$

$$y = y^4$$

$$0 = y^4 - y$$

$$0 = y(y^3 - 1)$$

$$y = 0, 1$$

But y-axis $\Rightarrow dy$
Right $\rightarrow y = x^2 \rightarrow x = \sqrt{y}$
Left $\rightarrow y = \sqrt{x} \rightarrow x = y^2$

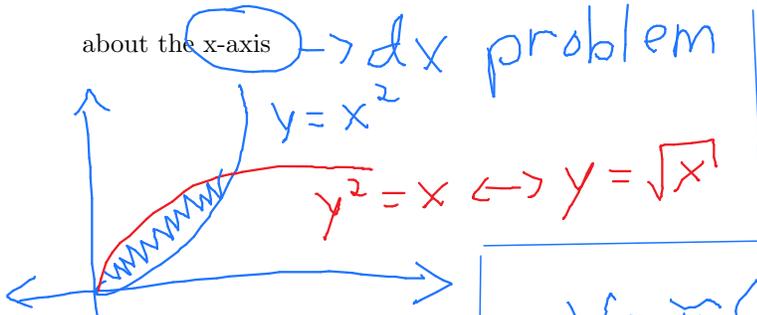
Volume =

$$\pi \int_0^1 [(\sqrt{y})^2 - (y^2)^2] dy$$

47. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = x^2, \text{ and } y^2 = x$$

about the x-axis $\rightarrow dx$ problem



Bounds

$$x^2 = \sqrt{x}$$

$$x^4 = x$$

$$x^4 - x = 0$$

$$x(x^3 - 1) = 0$$

$$x = 0, 1$$

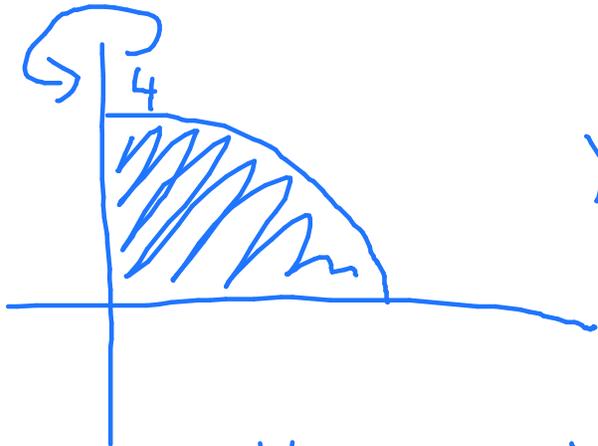
$$V = \pi \int_0^1 (\sqrt{x})^2 - (x^2)^2 dx$$

Volume =

$$\pi \int_0^1 (x - x^4) dx$$

48. Set up the integral that computes the **VOLUME** of the region generated by revolving the region in Quadrant I bounded by the following curves about the y -axis using the disk/washer method. $\Rightarrow dy$

$$y = 4 - x^2, \quad y = 0 \quad \text{and} \quad x = 0$$

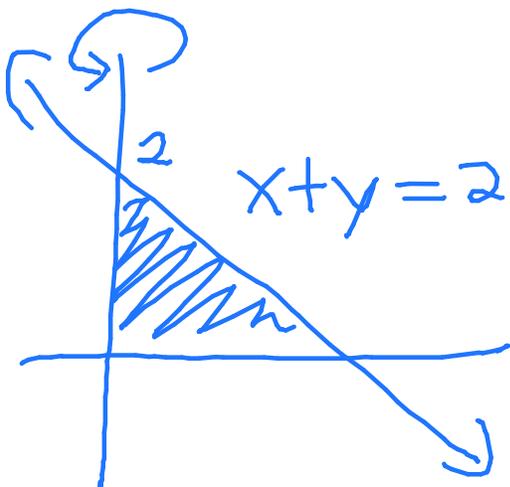


$$\begin{aligned} \Downarrow \\ y + x^2 &= 4 \\ x^2 &= 4 - y \\ x &= \sqrt{4 - y} \end{aligned}$$

$$V = \pi \int_0^4 (\sqrt{4-y})^2 dy$$

Volume = $\pi \int_0^4 (4-y) dy$

49. Find the volume of the solid generated by revolving the region bounded by $x + y = 2$ in Quadrant I about the y -axis. $\Rightarrow dy$



$$x + y = 2 \rightarrow x = 2 - y$$

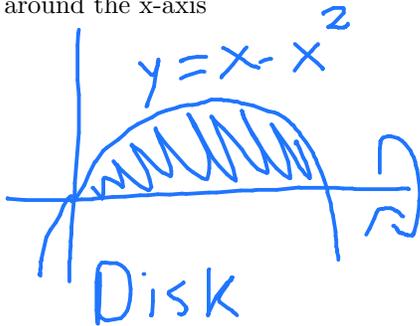
$$\begin{aligned} V &= \pi \int_0^2 (2-y)^2 dy \\ &= \pi \int_0^2 (y-2)^2 dy \\ &= \pi \left[\frac{(y-2)^3}{3} \right]_0^2 \\ &= \pi \left[\frac{(2-2)^3}{3} - \frac{(0-2)^3}{3} \right] \end{aligned}$$

Volume = $8\pi/3$

50. Find the **VOLUME** of the region bounded by

$$y = x - x^2, \text{ and } y = 0$$

around the x-axis



Bounds:

$$\begin{aligned} x - x^2 &= 0 \\ x(1-x) &= 0 \\ x &= 0, 1 \end{aligned}$$

$$\begin{aligned} V &= \pi \int_0^1 (x - x^2)^2 dx \\ &= \pi \int_0^1 (x^2 - 2x^3 + x^4) dx \\ &= \pi \left[\frac{x^3}{3} - \frac{2x^4}{4} + \frac{x^5}{5} \right]_0^1 \\ &= \frac{\pi}{30} \end{aligned}$$

Volume =

$\pi/30$

51. Find the **VOLUME** of the solid generate by revolving the given region about the x-axis $\rightarrow dx$

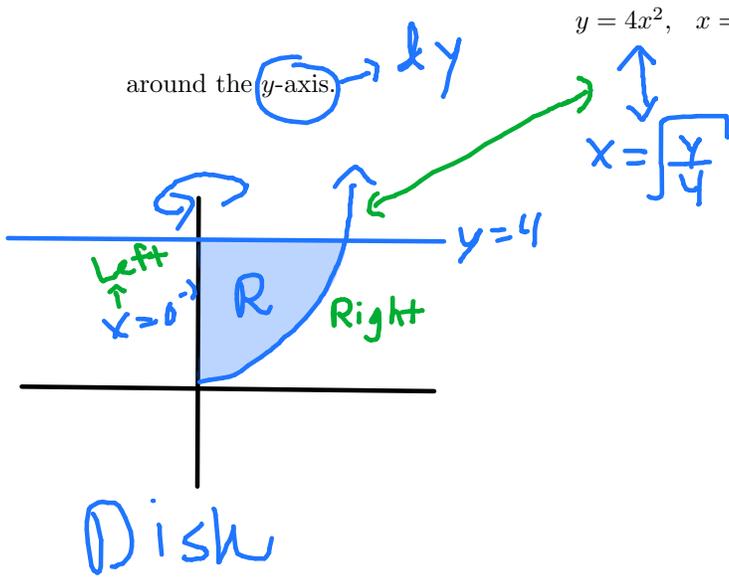
$$y = 8\sqrt{x}, \quad y = 0, \quad x = 3, \quad x = 6$$

$$\begin{aligned} V &= \pi \int_3^6 (8\sqrt{x})^2 dx \\ &= \pi \int_3^6 64x dx \\ &= \pi \left[\frac{64x^2}{2} \right]_3^6 \\ &= \pi \left[32x^2 \right]_3^6 \\ &= 264\pi \end{aligned}$$

Volume =

264π

52. Find the **VOLUME** of the region bounded by



$$\begin{aligned}
 V &= \pi \int_0^4 \left(\sqrt{\frac{y}{4}}\right)^2 dy \\
 &= \pi \int_0^4 \frac{y}{4} dy \\
 &= \frac{\pi}{4} \cdot \frac{y^2}{2} \Big|_0^4 \\
 &= \frac{\pi}{8} \cdot 16 \\
 &= 2\pi
 \end{aligned}$$

Volume = 2π

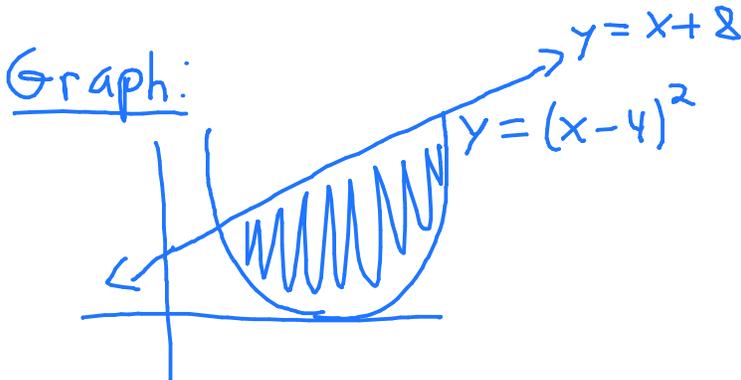
53. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = x + 8, \text{ and } y = (x - 4)^2$$

about the x -axis

Bounds:

$$\begin{aligned}
 x + 8 &= (x - 4)^2 \\
 x + 8 &= x^2 - 8x + 16 \\
 0 &= x^2 - 9x + 8 \\
 0 &= (x - 8)(x - 1) \\
 x &= 1, 8
 \end{aligned}$$



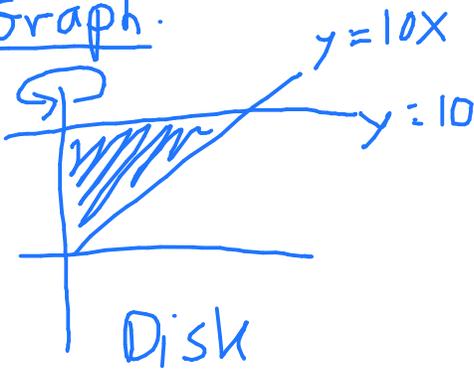
Volume = $\pi \int_1^8 [(x + 8)^2 - (x - 4)^4] dx$

54. Find the **VOLUME** of the region bounded by

$$y = 10x, \quad x = 0, \quad y = 10$$

around the y -axis

Graph:



But y -axis \Rightarrow dy problem
 $y = 10x$
 $\frac{y}{10} = x$

$$\begin{aligned} V &= \pi \int_0^{10} \left(\frac{y}{10}\right)^2 dy \\ &= \pi \int_0^{10} \frac{y^2}{100} dy \\ &= \frac{\pi}{100} \left(\frac{y^3}{3}\right) \Big|_0^{10} \\ &= \frac{10\pi}{3} \end{aligned}$$

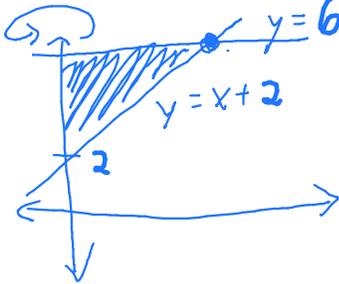
Volume =

$$\boxed{\frac{10\pi}{3}}$$

55. Find the **VOLUME** of the solid generated by rotating the region bounded by

$$y = x + 2, \quad x = 0, \quad y = 6 \quad \rightarrow \quad x = y - 2$$

around the y -axis \rightarrow dy problem.



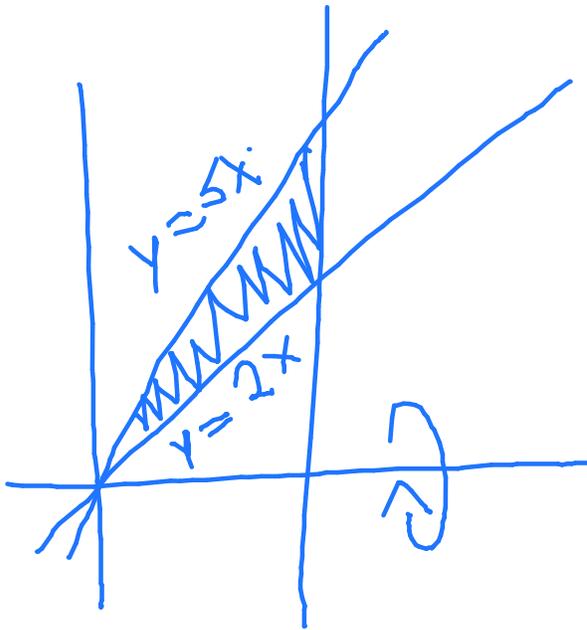
$$\begin{aligned} V &= \pi \int_2^6 (y-2)^2 dy \\ &= \pi \int_2^6 (y^2 - 4y + 4) dy \\ &= \pi \left(\frac{y^3}{3} - \frac{4y^2}{2} + 4y\right) \Big|_2^6 \end{aligned}$$

Volume =

$$\boxed{\frac{64\pi}{3}}$$

56. Find the volume of the solid generated by revolving the region bounded by the following curves about the x -axis. $\Rightarrow dx$

$$y = 2x, \quad y = 5x, \quad \text{and } x = 1$$



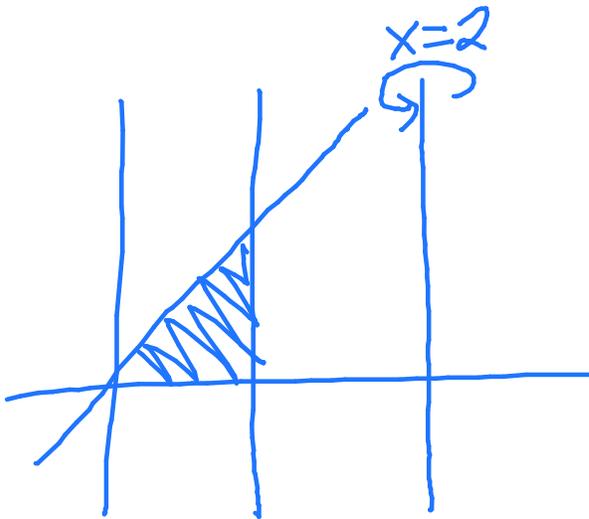
$$\begin{aligned} V &= \pi \int_0^1 (5x)^2 - (2x)^2 dx \\ &= \pi \int_0^1 25x^2 - 4x^2 dx \\ &= \pi \int_0^1 21x^2 dx \\ &= \pi \left[\frac{21}{3} x^3 \right]_0^1 = 7\pi \end{aligned}$$

Volume =

$$\boxed{7\pi}$$

57. Find the volume of the solid generated by revolving the region bounded by the following curves about the line $x = 2$. $\Rightarrow dx$

$$y = 2x, \quad y = 0, \quad \text{and } x = 1$$



$$\begin{aligned} \downarrow \\ x = \frac{y}{2} \end{aligned} \quad \begin{aligned} V &= \pi \int_0^1 (2 - \frac{y}{2})^2 - (2-1)^2 dy \\ &= \pi \int_0^1 (4 - 2y + \frac{y^2}{4}) - 1 dy \\ &= \pi \int_0^1 3 - 2y + \frac{y^2}{4} dy \\ &= \pi \left(3y - \frac{2y^2}{2} + \frac{1}{4} \cdot \frac{y^3}{3} \right) \Big|_0^1 \\ &= \frac{13\pi}{6} \end{aligned}$$

Volume =

$$\boxed{13\pi/6}$$

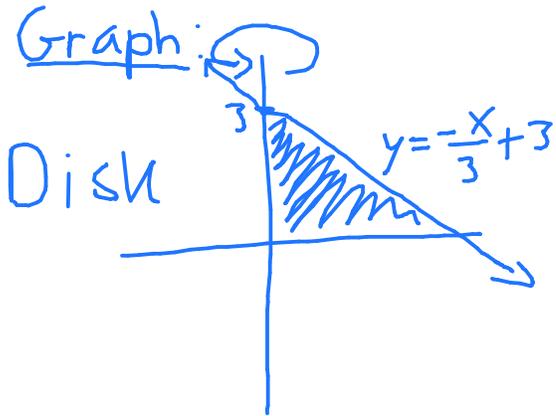
58. Find the **VOLUME** of the region bounded by

$$x + 3y = 9, \quad x = 0, \quad y = 0$$

around the y-axis

$$\begin{aligned} x + 3y &= 9 \\ 3y &= -x + 9 \\ y &= -\frac{x}{3} + 3 \end{aligned}$$

$$\begin{aligned} V &= \pi \int_0^3 (9 - 3y)^2 dy \\ &= \pi \int_0^3 (81 - 54y + 9y^2) dy \\ &= \pi \left(81y - 27y^2 + 3y^3 \right) \Big|_0^3 \\ &= 81\pi \end{aligned}$$



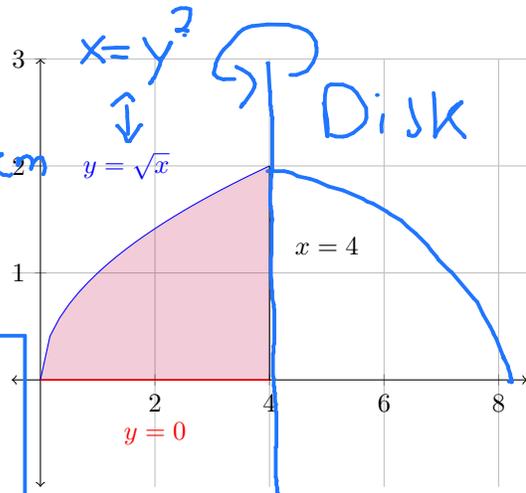
But y-axis \Rightarrow dy
 So $x + 3y = 9$
 $x = 9 - 3y$

Volume = $\boxed{81\pi}$

59. Let R be the region shown to the right. Set up the integral that computes the **VOLUME** as R is rotated around the line $x = 4$.

DON'T COMPUTE IT!!!

\rightarrow dy problem



Volume = $\boxed{\pi \int_0^2 (y^2 - 4)^2 dy}$

60. SET-UP using the washer method. the VOLUME of the region bounded by

$$y = x^2, \quad y = 2x$$

around the x-axis $\rightarrow dx$

(A) $\pi \int_0^2 (2x - x^2)^2 dx$

(B) $\pi \int_0^2 (4x^2 - x^4) dx$

(C) $\pi \int_0^2 (2x - x^2) dx$

(D) $\pi \int_0^2 (x^2 - 2x) dx$

(E) $\pi \int_0^2 (x^4 - 4x^2) dx$

(F) $2\pi \int_0^2 (x^3 - 2x^2) dx$

Note the bounds for all choices are the same.

Test Pt: $x=1$

$y = x^2 \rightarrow y = 1 \rightarrow$ Bottom

$y = 2x \rightarrow y = 2 \rightarrow$ Top

$$V = \pi \int_0^2 (2x)^2 - (x^2)^2 dx$$

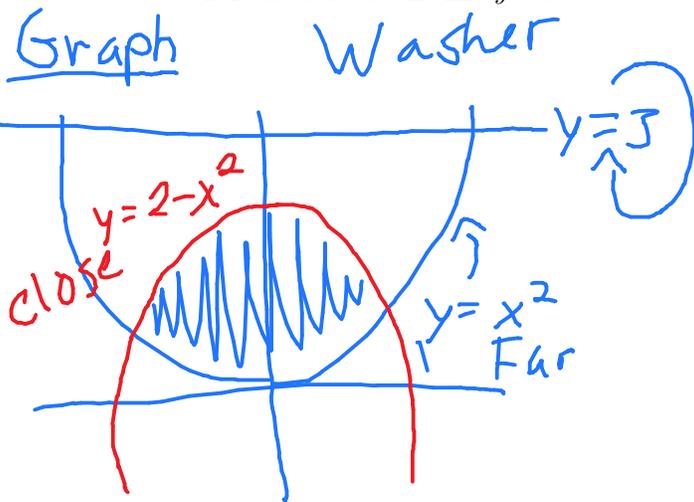
$$= \pi \int_0^2 4x^2 - x^4 dx$$

61. Set up the integral needed to find the volume of the solid obtained when the region bounded by

$$y = 2 - x^2 \quad \text{and} \quad y = x^2$$

is rotated about the line $y = 3$.

$y = 3 \Rightarrow$ dy problem



Bounds: $2 - x^2 = x^2$

$$2 = 2x^2$$

$$1 = x^2$$

$$x = \pm 1$$

$$\text{Volume} = \pi \int_{-1}^1 (2 - x^2 - 3)^2 - (x^2 - 3)^2 dx$$

62. SET-UP using the disk/washer method. the VOLUME of the region bounded by

$$y = 3x, \quad x = 0, \quad y = 27$$

Disk

around the line $y = 27 \rightarrow dx$

(A) $\pi \int_0^{27} (729 - 162x + 9x^2) dx$

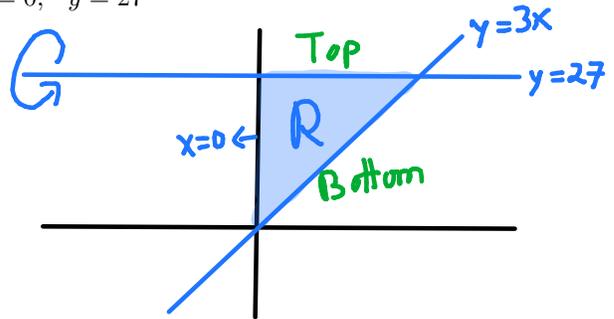
(B) $\pi \int_0^{27} 9x^2 dx$

(C) $\pi \int_0^9 9x^2 dx$

(D) $\pi \int_0^9 (9x^2 - 162x) dx$

(E) $\pi \int_0^{27} (729 - 9x^2) dx$

(F) $\pi \int_0^9 (729 - 162x + 9x^2) dx$



Bound: $\begin{cases} 3x = 27 \\ x = 9 \end{cases}$

$$\begin{aligned} V &= \pi \int_0^9 (3x - 27)^2 dx \\ &= \pi \int_0^9 (9x^2 - 162x + 729) dx \end{aligned}$$