

Please show **all** your work! Answers without supporting work will not be given credit.
Write answers in spaces provided.

Name: _____

1. **SET-UP using the Shell method**, the integral that computes the **VOLUME** of the region in quadrant I enclosed by the region defined by a triangle with vertices at (1,0), (6,0), and (6,10) about the y-axis.

(A) $\int_1^6 5\pi * (x^2 - x) dx$

(B) $\int_1^6 4\pi * (x^2) dx$

(C) $\int_1^6 4\pi * (x^2 - x) dx$

(D) $\int_1^6 2\pi * (x^2 - x) dx$

(E) $\int_1^6 5\pi * (x^2) dx$

(F) $\int_1^6 2\pi * (x^2) dx$

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2. Find the **VOLUME** of the region bounded by

$$y = 3x^2, \quad x = 0, \quad y = 27$$

around the line $y = 27$

Volume = _____

3. Find the volume of the solid obtained by revolving the region enclosed by the following curves about the x -axis using cylindrical shells.

$$x = 4y - y^2, \quad \text{and} \quad x = 0$$

Volume = _____

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4. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = 2 - x^2, \quad \text{and} \quad y = x^2$$

about the y-axis.

Volume = _____

5. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = 3\sqrt{x}, \quad \text{and} \quad y = x$$

about the $x = 12$.

Volume = _____

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6. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = x, \quad \text{and} \quad y = x^2$$

about the line $x = -2$.

Volume = _____

7. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = 7x^2, \quad y = 0 \quad \text{and} \quad x = 2$$

about the line $x = 3$.

Volume = _____

-
8. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$x = y^2 + 1, \text{ and } x = 2$$

about the line $y = -2$.

Volume = _____

9. The rate of change of the population $n(t)$ of a sample of bacteria is directly proportional to the number of bacteria present, so $N'(t) = kN$, where time t is measured in minutes. Initially, there are 210 bacteria present. If the number of bacteria after 5 hours is 360, find the growth rate k in terms of minutes. Round to four decimal places.

$k =$ _____

10. Let y denote the mass of a radioactive substance at time t . Suppose this substance obeys the equation

$$y' = -18y$$

Assume that initially, the mass of the substance is $y(0) = 20$ grams. At what time t in hours does half the original mass remain? Round your answer to 3 decimal places.

$$t = \underline{\hspace{10cm}}$$

11. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{3x^2}{y}$$

$$y = \underline{\hspace{10cm}}$$

12. Find the general solution to the differential equation:

$$\frac{dy}{dx} = 5y$$

$$y = \underline{\hspace{10cm}}$$

13. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$y = \underline{\hspace{10cm}}$$

14. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} - 15y = 0$$

$$y = \underline{\hspace{10cm}}$$

15. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = \frac{3}{y}$$

$$y = \underline{\hspace{10cm}}$$

16. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = 3x^2y$$

$$y = \underline{\hspace{10cm}}$$

17. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = 16e^{2x-y}$$

$$y = \underline{\hspace{10cm}}$$

18. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{3x + 2}{2y} \quad \text{and} \quad y(0) = 4$$

$y =$ _____

19. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{5y}{6x + 3} \quad \text{and} \quad y(0) = 1$$

$y =$ _____

20. Find the particular solution to the given differential equation.

$$\frac{dy}{dx} = 3x^2(9 + y), \quad y(0) = 5$$

$y =$ _____

21. Consider the following IVP:

$$\frac{dy}{dx} = 11x^2e^{-x^3} \text{ where } y = 10 \text{ when } x = 2$$

Find the value of the integration constant, C .

$$C = \underline{\hspace{15em}}$$

22. Find the particular solution to the given differential equation if $y(2) = 3$

$$\frac{dy}{dx} = \frac{x}{y^2}$$

$$y = \underline{\hspace{15em}}$$

23. Calculate the constant of integration, C , for the given differential equation.

$$\frac{dy}{dx} = \frac{7x^3}{6y}, \quad y(1) = 2$$

$C =$ _____

24. The volume of an object $V(t)$ in cubic millimeter at any time t in seconds changes according to the model

$$\frac{dV}{dt} = \cos\left(\frac{t}{10}\right),$$

where $V(0) = 5$. Find the volume of the object at $t = 3$ seconds. Round to 4 decimal places.

$V(3) =$ _____

25. What is the **integrating factor** of the following differential equation?

$$2y' + \left(\frac{6}{x}\right)y = 10\ln(x)$$

$$u(x) = \underline{\hspace{10cm}}$$

26. What is the **integrating factor** of the following differential equation?

$$y' + \left(\frac{2x+3}{x}\right)y = 10\ln(x)$$

$$u(x) = \underline{\hspace{10cm}}$$

27. What is the **integrating factor** of the following differential equation?

$$x^8 y' - 14x^7 y = 32e^{7x}$$

$$u(x) = \underline{\hspace{10cm}}$$

28. What is the **integrating factor** of the following differential equation?

$$(x + 1) \frac{dy}{dx} - 2(x^2 + x)y = (x + 1)e^{x^2}$$

$$u(x) = \underline{\hspace{10cm}}$$

29. What is the **integrating factor** of the following differential equation?

$$y' + \cot(x) \cdot y = \sin^2(x)$$

$$u(x) = \underline{\hspace{10cm}}$$

30. What is the **integrating factor** of the following differential equation?

$$y' + \tan(x) \cdot y = \sec(x)$$

$$u(x) = \underline{\hspace{10cm}}$$

31. Find the general solution of the following differential equation.

$$\frac{dy}{dx} + (4x - 1)y = 8x - 2$$

$y =$ _____

32. Find the general solution of the following differential equation.

$$\frac{dy}{dx} + \frac{6y}{x} = x + 10$$

$y =$ _____

33. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = 6x^2(y + 4) \text{ and } y(0) = 3$$

$y =$ _____

34. Solve the initial value problem.

$$x^4 y' + 4x^3 \cdot y = 10x^9 \text{ with } f(1) = 23$$

$y =$ _____

35. Find the general solution for the differential equation.

$$x \frac{dy}{dx} + 5y = 12x^2$$

$y =$ _____

36. Find the value of the constant of integration, C , for the particular solution to the following differential equation.

$$(x - 5)y' + y = x^2 + 14, \quad y(3) = 12$$

$C =$ _____

37. (a) Use summation notation to write the series in compact form.

$$1 - 0.6 + 0.36 - 0.216 + \dots$$

Answer: _____

(b) Use the sum from (a) and compute the sum.

Answer: _____

38. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$$

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n = \underline{\hspace{10em}}$$

39. Express the repeating decimal $7.33333\dots$ as a sum.

$$7.33333\dots = \underline{\hspace{10cm}}$$

40. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} 6 \left(-\frac{1}{9}\right)^n$$

$$\sum_{n=0}^{\infty} 6 \left(-\frac{1}{9}\right)^n = \underline{\hspace{10cm}}$$

41. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{7}{4^n} \right)$$

$$\sum_{n=0}^{\infty} \left(\frac{7}{4^n} \right) = \underline{\hspace{10cm}}$$

42. Compute

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n}$$

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n} = \underline{\hspace{10cm}}$$

45. Evaluate the sum of the following infinite series.

$$\sum_{n=1}^{\infty} \frac{4(3)^{n-1}}{5^n}$$

Answer: _____

46. Evaluate the sum of the following infinite series.

$$\sum_{n=1}^{\infty} \left(\frac{3^{n-1}}{4^n} + \frac{(-1)^{n+1}}{9^n} \right)$$

Answer: _____

47. Find the radius of convergence for the power series shown below.

$$f(x) = \frac{x}{6x + 13}$$

$R =$ _____

48. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3(-2x)^n$$

$R =$ _____

49. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3(7x^2)^n$$

$$R = \underline{\hspace{10cm}}$$

50. Express $f(x) = \frac{3}{1+2x}$ as a power series and determine its radius of convergence.

$$\frac{3}{1+2x} = \underline{\hspace{10cm}}$$

$$R = \underline{\hspace{10cm}}$$

51. Express $f(x) = \frac{x}{4 + 3x^2}$ as a power series.

$$\frac{x}{4 + 3x^2} = \underline{\hspace{10cm}}$$

52. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$\int \sin(x^{3/2}) dx$$

$$\int \sin(x^{3/2}) dx = \underline{\hspace{10cm}}$$

53. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$\int e^{-3x} dx$$

$$\int e^{-3x} dx = \underline{\hspace{10cm}}$$

54. Evaluate the following indefinite integral as a series. [Leave your answer as a sum.]

$$\int 5e^{5x^3} dx$$

$$\int 5e^{5x^3} dx = \underline{\hspace{10cm}}$$

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55. Use the first three terms of the powers series representation of the $f(x) = \frac{3x}{10 + 2x}$ to estimate $f(0.5)$.
Round to 4 decimal places.

$$f(0.5) \approx \underline{\hspace{10em}}$$

56. Evaluate the indefinite integral as a power series.

$$\int \frac{x}{3 + 5x^2} dx$$

Answer: _____

57. Use a power series to approximate the definite integral using the first 3 terms of the series.

$$\int_0^{0.24} \frac{x}{5+x^6} dx$$

$$\int_0^{0.24} \frac{x}{5+x^6} dx \approx \underline{\hspace{10em}}$$

-
58. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.11} \frac{1}{1+x^4} dx$$

$$\int_0^{0.11} \frac{1}{1+x^4} dx \approx \underline{\hspace{10em}}$$

59. Use a power series to approximate the definite integral using the first 3 terms of the series. Round to 5 decimal places.

$$\int_0^{0.23} e^{-x^2} dx$$

$$\int_0^{0.23} e^{-x^2} dx \approx \underline{\hspace{10em}}$$

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60. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.45} 4x \cos(\sqrt{x}) dx$$

$$\int_0^{0.45} 4x \cos(\sqrt{x}) dx \approx \underline{\hspace{4cm}}$$

61. Use the first five terms of the Macluarin series for $f(x) = \ln(1 + x)$ to evaluate $\ln(1.44)$. Round to 5 decimal places.

$$\ln(1.44) \approx \underline{\hspace{4cm}}$$

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62. Use the first 4 terms of the Macluarin series for $f(x) = \sin(x)$ to evaluate $\sin(0.75)$. Round to 5 decimal places.

$$\sin(0.75) \approx \underline{\hspace{10em}}$$

63. Given $f(x, y) = 3x^3y^2 - x^2y^{1/3}$, evaluate $f(3, -8)$.

$$f(3, -8) = \underline{\hspace{10em}}$$

64. Find the domain of

$$f(x, y) = \frac{-5x}{\sqrt{x + 9y + 1}}$$

$$\text{Domain} = \underline{\hspace{10em}}$$

65. Find the domain of

$$f(x, y) = \frac{\sqrt{x + y - 1}}{\ln(y - 11) - 9}$$

Domain = _____

66. Find the domain of

$$f(x, y) = \frac{\ln(x^2 - y + 3)}{\sqrt{x - 6}}$$

Domain = _____

67. Describe the indicated level curves $f(x, y) = C$

$$f(x, y) = \ln(x^2 + y^2) \quad C = \ln(36)$$

- (a) Parabola with vertices at $(0, 0)$
- (b) Circle with center at $(0, \ln(36))$ and radius 6
- (c) Parabola with vertices at $(0, \ln(36))$
- (d) Circle with center at $(0, 0)$ and radius 6
- (e) Increasing Logarithm Function

68. What do the level curves for the following function look like?

$$f(x, y) = \ln(y - e^{5x})$$

- (a) Increasing exponential functions
- (b) Rational Functions with x-axis symmetry
- (c) Natural logarithm functions
- (d) Decreasing exponential functions
- (e) Rational Functions with y-axis symmetry

69. What do the level curves for the following function look like?

$$f(x, y) = \sqrt{y + 4x^2}$$

- (a) Lines
- (b) Parabolas
- (c) Circles
- (d) Point at the origin
- (e) Ellipses
- (f) Hyperbolas

70. What do the level curves for the following function look like?

$$f(x, y) = \cos(y + 4x^2)$$

- (a) Lines
- (b) Parabolas
- (c) Circles
- (d) Point at the origin
- (e) Ellipses
- (f) Hyperbolas

71. What do the level curves for the following function look like?

$$f(x, y) = \ln(8y - 5x^2)$$

- (a) logarithmic curves
- (b) lines
- (c) a point at the origin
- (d) circles
- (e) hyperbolas
- (f) parabolas

72. For the following function $f(x, y)$, evaluate $f_y(-2, -3)$.

$$f(x, y) = 8x^4y^5 + 3x^3 - 12y^2$$

$$f_y(-2, -3) = \underline{\hspace{10em}}$$

73. Compute $f_x(6, 5)$ when

$$f(x, y) = \frac{(6x - 6y)^2}{\sqrt{y^2 - 1}}$$

$$f_x(6, 5) = \underline{\hspace{10cm}}$$

74. Given the function $f(x, y) = \sin(x^2y)$, evaluate $f_x\left(\frac{1}{2}, \pi\right)$.

$$f_x\left(\frac{1}{2}, \pi\right) = \underline{\hspace{10cm}}$$

75. Find the first order partial derivatives of

$$f(x, y) = \frac{3x^2y^3}{(y-1)^2}$$

$$f_x(x, y) = \underline{\hspace{10cm}}$$

$$f_y(x, y) = \underline{\hspace{10cm}}$$

76. Find the first order partial derivatives of

$$f(x, y) = x \sin(xy)$$

$$f_x(x, y) = \underline{\hspace{10cm}}$$

$$f_y(x, y) = \underline{\hspace{10cm}}$$

77. Find the first order partial derivatives of $f(x, y) = (xy - 1)^2$

$$f_x(x, y) = \underline{\hspace{10cm}}$$

$$f_y(x, y) = \underline{\hspace{10cm}}$$

78. Find the first order partial derivatives of $f(x, y) = xe^{x^2+xy+y^2}$

$$f_x(x, y) = \underline{\hspace{10cm}}$$

$$f_y(x, y) = \underline{\hspace{10cm}}$$

79. Find the first order partial derivatives of $f(x, y) = -7 \tan(x^7 y^8)$

$$f_x(x, y) = \underline{\hspace{10cm}}$$

$$f_y(x, y) = \underline{\hspace{10cm}}$$

80. Find the first order partial derivatives of $f(x, y) = y \cos(x^2 y)$

$$f_x(x, y) = \underline{\hspace{10cm}}$$

$$f_y(x, y) = \underline{\hspace{10cm}}$$

81. Find the first order partial derivatives of $f(x, y) = xe^{xy}$

$$f_x(x, y) = \underline{\hspace{10cm}}$$

$$f_y(x, y) = \underline{\hspace{10cm}}$$