

Please show **all** your work! Answers without supporting work will not be given credit.  
Write answers in spaces provided.

## Solutions

Name: \_\_\_\_\_

1. **SET-UP using the Shell method**, the integral that computes the **VOLUME** of the region in quadrant I enclosed by the region defined by a triangle with vertices at  $(1,0)$ ,  $(6,0)$ , and  $(6,10)$  about the  $y$ -axis.

(A)  $\int_1^6 5\pi * (x^2 - x) dx$

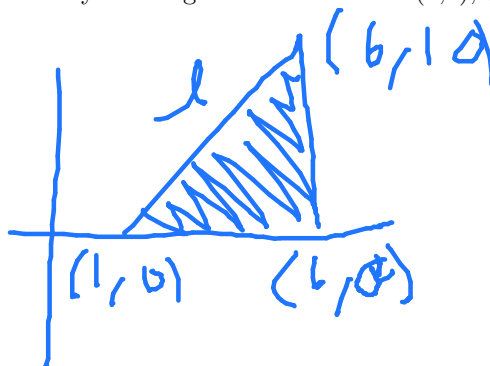
(B)  $\int_1^6 4\pi * (x^2) dx$

(C)  $\int_1^6 4\pi * (x^2 - x) dx$

(D)  $\int_1^6 2\pi * (x^2 - x) dx$

(E)  $\int_1^6 5\pi * (x^2) dx$

(F)  $\int_1^6 2\pi * (x^2) dx$



$$m = \frac{10-0}{6-1} = 2$$

$$l: y = x + b$$

Find  $b$  w/  $(1,0)$

$$0 = 2 + b \rightarrow b = -2$$

$$y = 2x - 2$$

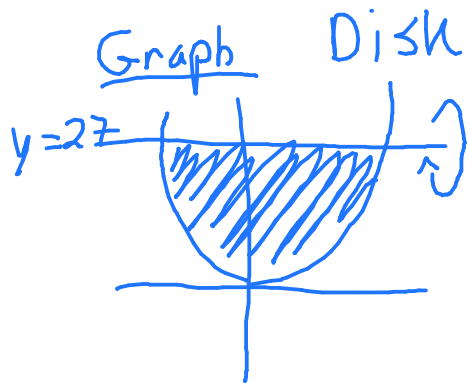
$$\begin{aligned} V &= 2\pi \int_1^6 x(2x-2) dx \\ &= \int_1^6 4\pi(x^2 - x) dx \end{aligned}$$

Note the bounds are all the same.

2. Find the **VOLUME** of the region bounded by

$$y = 3x^2, \quad x = 0, \quad y = 27$$

around the line  $y = 27$



$y = 27 \Rightarrow dx$  problem

Bounds: Given  $x = 0$

$$27 = 3x^2$$

$$9 = x^2 \rightarrow x = 3$$

$$\begin{aligned} V &= \pi \int_0^3 (3x^2 - 27)^2 dx \\ &= \pi \int_0^3 (9x^4 - 162x^2 + 729) dx \\ &= \pi \left( \frac{9x^5}{5} - 54x^3 + 729x \right) \Big|_0^3 \\ &= 11664.4\pi \end{aligned}$$

$$\boxed{\frac{8322\pi}{5}}$$

Volume = \_\_\_\_\_

3. Find the volume of the solid obtained by revolving the region enclosed by the following curves about the  $x$ -axis using cylindrical shells.

Bounds:

$$\begin{aligned} 0 &= 4y - y^2 \\ 0 &= y(4 - y) \\ y &= 0, 4 \end{aligned}$$

$$x = 4y - y^2, \quad \text{and} \quad x = 0$$

$$\begin{aligned} V &= 2\pi \int_0^4 y(4y - y^2) dy \\ &= 2\pi \int_0^4 (4y^2 - y^3) dy \\ &= 2\pi \left( \frac{4y^3}{3} - \frac{y^4}{4} \right) \Big|_0^4 \\ &= \frac{128}{3} \pi \end{aligned}$$

Volume =  $\boxed{\frac{128}{3} \pi}$  \_\_\_\_\_

4. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = 2 - x^2, \text{ and } y = x^2$$

about the **y-axis**.

+ ↓  
→ dx

Bounds:  $2 - x^2 = x^2$   
 $2 = 2x^2$   
 $1 = x^2$   
 $x = \pm 1$

$$V = 2\pi \int_{-1}^1 x(2 - x^2 - x^2) dx$$

Test Pt:  $x = 0$

$$y = 2 - x^2 \rightarrow y = 2 \rightarrow \text{Top}$$

$$y = x^2 \rightarrow y = 0 \rightarrow \text{Bottom}$$

Volume =

$$2\pi \int_{-1}^1 x(2 - 2x^2) dx$$

5. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = 3\sqrt{x}, \text{ and } y = x$$

about the **x = 12**.

+ ↓  
→ dx

Bounds

$$3\sqrt{x} = x$$

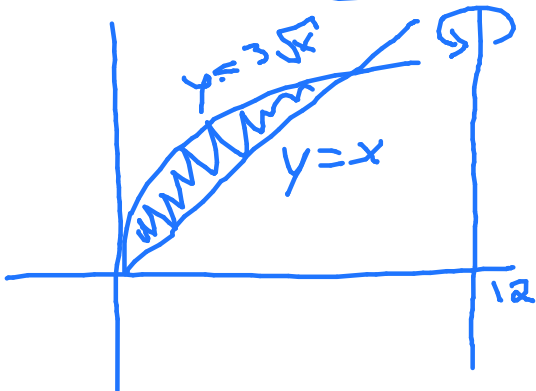
$$9x = x^2$$

$$9x - x^2 = 0$$

$$x(9 - x) = 0$$

$$x = 0, 9$$

★ Note  $x = 12$  is on the right of our region.



$$V = 2\pi \int_0^9 (12 - x)(3\sqrt{x} - x) dx$$

Volume =

6. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = x, \text{ and } y = x^2$$

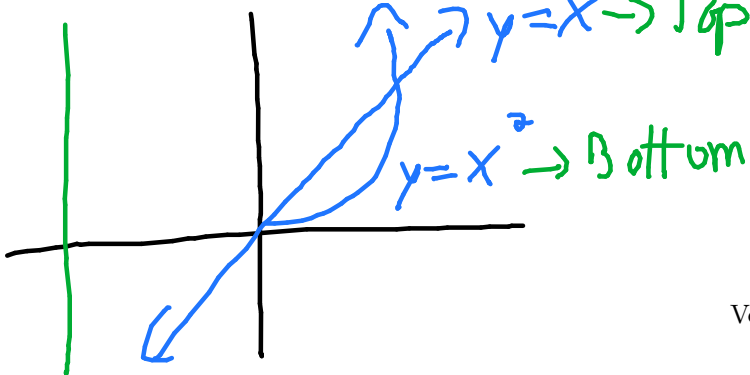
about the line  $x = -2$ .

Since  $x = -2$  is on the left of our region

$$V = 2\pi \int_0^1 (x - (-2)) [x - x^2] dx$$

Bounds:  $x = x^2$   
 $x - x^2 = 0$   
 $x(1-x) = 0$   
 $x = 0, 1$

$$x = -2$$



Volume = \_\_\_\_\_

$$2\pi \int_0^1 (x+2)[x-x^2] dx$$

7. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = 7x^2, \quad y = 0 \text{ and } x = 2$$

about the line  $x = 3$ .

$$V = 2\pi \int_0^2 ( \quad ) (7x^2) dx$$

Since  $x = 3$  is larger than the bounds,

$$V = 2\pi \int_0^2 (3-x)(7x^2) dx$$

Volume = \_\_\_\_\_

8. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$x = y^2 + 1, \text{ and } x = 2$$

about the line  $y = -2$ .

$dy$

Since  $y = -2$  is smaller than the bounds

$$V = 2\pi \int_{-1}^1 (y - (-2))(2 - (y^2 + 1)) dy$$

Bounds:  $y^2 + 1 = 2$   
 $y^2 = 1$   
 $y = \pm 1$

Test Pti  $y = 0$

$$x = y^2 + 1 \rightarrow x = 1 \rightarrow \text{Left}$$

$$x = 2 \rightarrow x = 2 \rightarrow \text{Right}$$

$$2\pi \int_{-1}^1 (y + 2)(2 - (y^2 + 1)) dy$$

Volume = \_\_\_\_\_

9. The rate of change of the population  $n(t)$  of a sample of bacteria is directly proportional to the number of bacteria present, so  $N'(t) = kN$ , where time  $t$  is measured in minutes. Initially, there are 210 bacteria present. If the number of bacteria after 5 hours is 360, find the growth rate  $k$  in terms of minutes. Round to four decimal places.

Recall  $N' = kN \rightarrow N = Ce^{kt}$

$N(0) = 210$ :  $210 = Ce^{k \cdot 0}$

$$210 = C \rightarrow N = 210e^{kt}$$

$N(5) = 360$ :  $360 = 210e^{k \cdot 5}$

$$\frac{12}{7} = e^{5k}$$

$$\ln\left(\frac{12}{7}\right) = 5k$$

$k =$  \_\_\_\_\_

$$\frac{1}{5} \ln\left(\frac{12}{7}\right)$$

10. Let  $y$  denote the mass of a radioactive substance at time  $t$ . Suppose this substance obeys the equation

$$y' = -18y$$

Assume that initially, the mass of the substance is  $y(0) = 20$  grams. At what time  $t$  in hours does half the original mass remain? Round your answer to 3 decimal places.

$$\begin{aligned} y' = -18y &\Rightarrow y = Ce^{-18t} \\ y(0) = 20 &\Rightarrow 20 = Ce^{-18(0)} \\ 20 = C &\Rightarrow y = 20e^{-18t} \end{aligned}$$

We want solve  $\frac{1}{2}(20) = y(t)$  for  $t$ .

$$10 = 20e^{-18t}$$

$$\frac{1}{2} = e^{-18t}$$

$$\ln\left(\frac{1}{2}\right) = -18t$$

$$\frac{\ln\left(\frac{1}{2}\right)}{-18} = t$$

$t =$

0.039

11. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{3x^2}{y}$$

Rewrite:  $y dy = 3x^2 dx$

$$\int y dy = \int 3x^2 dx$$

$$\frac{y^2}{2} = x^3 + C$$

$$y^2 = 2x^3 + C$$

$$y = \pm \sqrt{2x^3 + C}$$

$y =$

$\pm \sqrt{2x^3 + C}$

12. Find the general solution to the differential equation:

$$\frac{dy}{dx} = 5y$$

Rewrite  $dy = 5y dx$   
 $\frac{dy}{y} = 5dx$   
 $\int \frac{dy}{y} = \int 5dx$   
 $\ln|y| = 5x + C$   
 $|y| = e^{5x+C}$   
 $\pm y = e^C e^{5x}$   
 $y = \pm e^C e^{5x}$   
 $y = C e^{5x}$

or memorize  
 $\frac{dy}{dx} = ky$   
 $\Rightarrow y = C e^{kx}$

$y =$   $C e^{5x}$

13. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{-x}{y}$$

Rewrite:  $y dy = -x dx$   
 $\int y dy = \int -x dx$   
 $\frac{y^2}{2} = -\frac{x^2}{2} + C$   
 $y^2 = -x^2 + C$   
 $y = \pm \sqrt{C - x^2}$

$y =$   $\pm \sqrt{C - x^2}$

14. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} - 15y = 0$$

Note there are 2 ways to do this problem.

- ① Separation of Variables
- ② First-order Linear Eqn

$$\ln|y| = 15t + C$$

$$y = e^{15t + C}$$

$$y = e^C e^{15t}$$

$$y = Ce^{15t}$$

By method 1,

$$\frac{dy}{dt} = 15y$$

$$\frac{dy}{y} = 15 dt$$

$$\int \frac{dy}{y} = \int 15 dt$$

$$y = \boxed{Ce^{15t}}$$

15. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = \frac{3}{y}$$

$$y dy = 3 dx$$

$$\int y dy = \int 3 dx$$

$$\frac{y^2}{2} = 3x + C$$

$$y^2 = 6x + 2C$$

$$y^2 = 6x + C$$

$$y = \pm \sqrt{6x + C}$$

$$y = \boxed{\pm \sqrt{6x + C}}$$

16. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{y} = 3x^2 dx \quad \frac{dy}{dx} = 3x^2 y$$

$$\int \frac{dy}{y} = \int 3x^2 dx$$

$$\ln|y| = x^3 + C$$

$$y = e^{x^3 + C}$$

$$y = e^C e^{x^3}$$

$$y = C e^{x^3}$$

y =

$$C e^{x^3}$$

17. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$dy = 16e^{2x} e^{-y} dx \quad \frac{dy}{dx} = 16e^{2x-y}$$

$$e^y dy = 16e^{2x} dx$$

$$\int e^y dy = \int 16e^{2x} dx$$

$$e^y = \frac{16}{2} e^{2x} + C$$

$$e^y = 8e^{2x} + C$$

$$y = \ln(8e^{2x} + C)$$

y =

$$\ln(8e^{2x} + C)$$

18. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{3x+2}{2y} \text{ and } y(0) = 4$$

$$2y dy = (3x+2) dx$$

$$\int 2y dy = \int (3x+2) dx$$

$$y^2 = \frac{3x^2}{2} + 2x + C$$

$$\text{So } y^2 = \frac{3x^2}{2} + 2x + 16$$

$$y = \pm \sqrt{\frac{3x^2}{2} + 2x + 16}$$

when  $y(0) = 4$

$$4^2 = 0 + 0 + C$$

$$16 = C$$

$$y = \pm \sqrt{\frac{3x^2}{2} + 2x + 16}$$

19. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{5y}{6x+3} \text{ and } y(0) = 1$$

$$\frac{dy}{y} = \frac{5}{6x+3} dx$$

$$\int \frac{dy}{y} = \int \frac{5}{6x+3} dx$$

$$\ln|y| = \frac{5}{6} \ln|6x+3| + C$$

When  $y(0) = 1$

$$1 = C \cdot |6(0)+3|^{5/6}$$

$$1 = C \cdot 3^{5/6}$$

$$C = 3^{-5/6}$$

$$y = \exp\left[\frac{5}{6} \ln|6x+3| + C\right]$$

$$y = e^C \exp\left[\ln|6x+3|^{5/6}\right]$$

$$y = C \cdot |6x+3|^{5/6}$$

$$y = 3^{-5/6} \cdot |6x+3|^{5/6}$$

20. Find the particular solution to the given differential equation.

$$\frac{dy}{dx} = 3x^2(9+y), \quad y(0) = 5$$

$$\frac{dy}{9+y} = 3x dx$$

$$\int \frac{dy}{9+y} = \int 3x dx$$

$$\ln|9+y| = \frac{3}{2}x^2 + C$$

$$|9+y| = \exp\left[\frac{3}{2}x^2 + C\right]$$

$$\pm(9+y) = e^C \exp\left[\frac{3}{2}x^2\right]$$

$$9+y = \pm e^C \exp\left[\frac{3}{2}x^2\right]$$

$$9+y = C \exp\left[\frac{3}{2}x^2\right]$$

$$y = C \exp\left[\frac{3}{2}x^2\right] - 9$$

when  $y(0) = 5$

$$y = C \exp\left[\frac{3}{2}x^2\right] - 9$$

$$5 = C \exp\left[\frac{3}{2}(0)^2\right] - 9$$

$$5 = C - 9$$

$$14 = C$$

$$\Rightarrow y = 14 \exp\left[\frac{3}{2}x^2\right] - 9$$

$$y = \boxed{14 \exp\left[\frac{3}{2}x^2\right] - 9}$$

21. Consider the following IVP:

$$\frac{dy}{dx} = 11x^2 e^{-x^3} \text{ where } y = 10 \text{ when } x = 2$$

Find the value of the integration constant,  $C$ .

$$\begin{aligned} dy &= 11x^2 e^{-x^3} dx \\ \int dy &= \int 11x^2 e^{-x^3} dx \\ u &= -x^3 \\ du &= -3x^2 dx \\ y &= \int -\frac{11}{3} e^u du \\ y &= -\frac{11}{3} e^{-x^3} + C \end{aligned}$$

When  $y=10$  and  $x=2$

$$\begin{aligned} 10 &= -\frac{11}{3} e^{-2^3} + C \\ 10 &= -\frac{11}{3} e^{-8} + C \\ C &= 10 + \frac{11}{3} e^{-8} \end{aligned}$$

$$C = \boxed{10 + \frac{11}{3} e^{-8}}$$

22. Find the particular solution to the given differential equation if  $y(2) = 3$

$$\frac{dy}{dx} = \frac{x}{y^2}$$

$$\begin{aligned} y^2 dy &= x dx \\ \int y^2 dy &= \int x dx \\ \frac{y^3}{3} &= \frac{x^2}{2} + C \\ \text{Find } C \text{ w/ } y(2) &= 3 \\ \frac{3^3}{3} &= \frac{2^2}{2} + C \\ 9 &= 2 + C \\ 7 &= C \end{aligned}$$

$$\begin{aligned} \frac{y^3}{3} &= \frac{x^2}{2} + 7 \\ y^3 &= \frac{3x^2}{2} + 21 \\ y &= \sqrt[3]{\frac{3x^2}{2} + 21} \end{aligned}$$

$$y = \boxed{\sqrt[3]{\frac{3x^2}{2} + 21}}$$

23. Calculate the constant of integration,  $C$ , for the given differential equation.

$$\frac{dy}{dx} = \frac{7x^3}{6y}, \quad y(1) = 2$$

Rewrite  $6y \, dy = 7x^3 \, dx$   
 $\int 6y \, dy = \int 7x^3 \, dx$   
 $3y^2 = \frac{7x^4}{4} + C$

Note we want  $C$  when  $y(1) = 2$

$$3(2)^2 = \frac{7(1)^4}{4} + C$$

$$12 = \frac{7}{4} + C$$

$$C = 41/4$$

$$C = \boxed{\frac{41}{4}}$$

24. The volume of an object  $V(t)$  in cubic millimeter at any time  $t$  in seconds changes according to the model

$$\frac{dV}{dt} = \cos\left(\frac{t}{10}\right),$$

where  $V(0) = 5$ . Find the volume of the object at  $t = 3$  seconds. Round to 4 decimal places.

Rewrite  $dV = \cos\left(\frac{t}{10}\right) dt$   
 $\int dV = \int \cos\left(\frac{t}{10}\right) dt$   
 $V = 10 \sin\left(\frac{t}{10}\right) + C$

Find  $C$  w/  $V(0) = 5$

$$5 = 10 \sin\left(\frac{0}{10}\right) + C$$

$$C = 5$$

So  $V = 10 \sin\left(\frac{t}{10}\right) + 5$

$$V(3) = 10 \sin\left(\frac{3}{10}\right) + 5$$

$$\approx 7.9552$$

$$V(3) =$$

$$\boxed{7.9552}$$

25. What is the **integrating factor** of the following differential equation?

$$\frac{2y' + \left(\frac{6}{x}\right)y}{2} = \frac{10 \ln(x)}{2}$$

$$y' + \frac{3}{x}y = 5 \ln x$$

$$P(x) = \frac{3}{x} \quad Q(x) = 5 \ln x$$

$$\begin{aligned} u(x) &= \exp\left[\int \frac{3}{x} dx\right] \\ &= \exp[3 \ln x] \\ &= \exp[\ln x^3] \\ &= x^3 \end{aligned}$$

$$\boxed{x^3}$$

$u(x) =$  \_\_\_\_\_

26. What is the **integrating factor** of the following differential equation?

$$y' + \left(\frac{2x+3}{x}\right)y = 10 \ln(x)$$

$$P(x) = \frac{2x+3}{x} \quad Q(x) = 10 \ln(x)$$

$$\begin{aligned} u(x) &= \exp\left[\int P(x) dx\right] \\ &= \exp\left[\int \frac{2x+3}{x} dx\right] \\ &= \exp\left[\int 2 + \frac{3}{x} dx\right] \\ &= \exp[2x + 3 \ln x] \end{aligned}$$

$$\begin{aligned} &= e^{2x+3 \ln x} \\ &= e^{2x} \cdot e^{3 \ln x} \\ &= e^{2x} \cdot e^{\ln x^3} \\ &= x^3 e^{2x} \end{aligned}$$

$$\boxed{x^3 e^{2x}}$$

$u(x) =$  \_\_\_\_\_

27. What is the **integrating factor** of the following differential equation?

$$\frac{x^8 y' - 14x^7 y}{x^8} = \frac{32e^{7x}}{x^8}$$

$$y' + \underbrace{\left(-\frac{14}{x}\right)}_P y = \underbrace{\frac{32e^{7x}}{x^8}}_Q$$

$$\begin{aligned} u(x) &= \exp\left[\int P(x) dx\right] \\ &= \exp\left[\int -\frac{14}{x} dx\right] \\ &= \exp[-14 \ln x] \\ &= \exp[\ln x^{-14}] \\ &= x^{-14} \\ &= \frac{1}{x^{14}} \end{aligned}$$

$u(x) =$

$\frac{1}{x^{14}}$

28. What is the **integrating factor** of the following differential equation?

$$\frac{(x+1) \frac{dy}{dx} - 2(x^2+x)y}{(x+1)} = \frac{(x+1)e^{x^2}}{(x+1)}$$

$$\frac{dy}{dx} - \frac{2x(x+1)}{(x+1)} y = e^{x^2}$$

$$\frac{dy}{dx} + (-2x) \cdot y = e^{x^2}$$

$$u(x) = \exp\left[\int P(x) dx\right]$$

$$= \exp\left[\int -2x dx\right]$$

$$= \exp[-x^2]$$

$u(x) =$

$e^{-x^2}$

29. What is the **integrating factor** of the following differential equation?

$$y' + \cot(x) \cdot y = \sin^2(x)$$

$$\begin{aligned} u(x) &= \exp\left[\int P(x) dx\right] \\ &= \exp\left[\int \cot x dx\right] \\ &= \exp\left[\int \frac{\cos x}{\sin x} dx\right] \end{aligned}$$

$$\begin{aligned} u &= \sin x \\ du &= \cos x dx \\ &= \exp\left[\int \frac{du}{u}\right] \\ &= \exp[\ln u] \end{aligned}$$

$$\begin{aligned} u(x) &= \exp[\ln \sin x] \\ &= \sin x \end{aligned}$$

$$u(x) = \boxed{\sin x}$$

30. What is the **integrating factor** of the following differential equation?

$$y' + \tan(x) \cdot y = \sec(x)$$

$$\begin{aligned} u(x) &= \exp\left[\int P(x) dx\right] \\ &= \exp\left[\int \tan x dx\right] \\ &= \exp\left[\int \frac{\sin x}{\cos x} dx\right] \end{aligned}$$

$$\begin{aligned} u &= \cos x \\ du &= -\sin x dx \\ &= \exp\left[-\int \frac{du}{u}\right] \\ &= \exp[-\ln u] \end{aligned}$$

$$\begin{aligned} u(x) &= \exp[-\ln(\cos x)] \\ &= \exp[\ln(\cos x)^{-1}] \\ &= (\cos x)^{-1} = \sec x \end{aligned}$$

$$u(x) = \boxed{\sec(x)}$$

31. Find the general solution of the following differential equation.

$$\frac{dy}{dx} + (4x - 1)y = 8x - 2$$

$$P(x) = 4x - 1 \quad Q(x) = 8x - 2$$

$$u(x) = \exp\left[\int (4x - 1) dx\right]$$

$$= \exp[2x^2 - x]$$

$$= e^{2x^2 - x}$$

$$y u(x) = \int Q(x) u(x) dx + C$$

$$y e^{2x^2 - x} = \int (8x - 2) e^{2x^2 - x} dx + C$$

$$u = 2x^2 - x$$

$$du = 4x - 1 dx$$

$$y e^{2x^2 - x} = \int \frac{8x - 2}{4x - 1} e^u du + C$$

$$y e^{2x^2 - x} = \int \frac{2(4x - 1)}{4x - 1} e^u du + C$$

$$y e^{2x^2 - x} = \int 2e^u du + C$$

$$y e^{2x^2 - x} = 2e^u + C$$

$$y e^{2x^2 - x} = 2e^{2x^2 - x} + C$$

Note there are 2 ways to do this problem.

- ① Separation of Variables
- ② First-order Linear Eqn

$$y = \frac{2e^{2x^2 - x} + C}{e^{2x^2 - x}}$$

$$y = 2 + Ce^{-(2x^2 - x)}$$

$$= 2 + Ce^{x - 2x^2}$$

y =

$$2 + Ce^{x - 2x^2}$$

32. Find the general solution of the following differential equation.

$$\frac{dy}{dx} + \frac{6y}{x} = x + 10$$

$$P(x) = \frac{6}{x} \quad Q(x) = x + 10$$

$$\begin{aligned} u(x) &= \exp\left[\int P(x) dx\right] \\ &= \exp\left[\int \frac{6}{x} dx\right] \\ &= \exp[6 \ln x] \\ &= \exp[\ln(x^6)] \\ &= x^6 \end{aligned}$$

$$y \cdot u(x) = \int Q(x)u(x) dx + C$$

$$y x^6 = \int (x+10)x^6 dx + C$$

$$y x^6 = \int (x^7 + 10x^6) dx + C$$

$$y x^6 = \frac{x^8}{8} + \frac{10x^7}{7} + C$$

$$y = \frac{x^2}{8} + \frac{10x}{7} + \frac{C}{x^6}$$

y =

$$\frac{x^2}{8} + \frac{10x}{7} + \frac{C}{x^6}$$

33. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = 6x^2(y+4) \text{ and } y(0) = 3$$

$$y' = 6x^2 y + 24x^2$$

$$y' - 6x^2 y = 24x^2$$

$$P(x) = -6x^2 \quad Q(x) = 24x^2$$

$$u(x) = \exp\left[\int -6x^2 dx\right]$$

$$= \exp[-2x^3]$$

$$= e^{-2x^3}$$

$$y \cdot u(x) = \int Q(x)u(x) dx + C$$

$$ye^{-2x^3} = \int 24x^2 e^{-2x^3} dx + C$$

$$u = -2x^3$$

$$du = -6x^2 dx$$

$$ye^{-2x^3} = \int -4e^u du + C$$

$$ye^{-2x^3} = -4e^u + C$$

$$ye^{-2x^3} = -4e^{-2x^3} + C$$

$$y = -4 + Ce^{2x^3}$$

$$\text{With } y(0) = 3$$

$$3 = -4 + Ce^{2 \cdot 0^3}$$

$$3 = -4 + C$$

$$7 = C$$

$$\text{So } y = -4 + 7e^{2x^3}$$

y =

$$-4 + 7e^{2x^3}$$

34. Solve the initial value problem.

$$x^4 y' + 4x^3 \cdot y = 10x^9 \text{ with } f(1) = 23$$

$$\frac{x^4 y' + 4x^3 y}{x^4} = \frac{10x^9}{x^4}$$

$$y' + \frac{4}{x} \cdot y = 10x^5$$

$$P(x) = \frac{4}{x} \quad Q(x) = 10x^5$$

$$u(x) = \exp\left[\int P(x) dx\right]$$

$$= \exp\left[\int \frac{4}{x} dx\right]$$

$$= \exp[4 \ln x]$$

$$= \exp[\ln x^4]$$

$$= x^4$$

$$y \cdot u(x) = \int Q(x)u(x) dx + C$$

$$y \cdot x^4 = \int 10x^5 x^4 dx + C$$

$$y \cdot x^4 = \int 10x^9 dx + C$$

$$y \cdot x^4 = x^{10} + C$$

$$y = \frac{x^{10}}{x^4} + \frac{C}{x^4}$$

$$y = x^6 + \frac{C}{x^4}$$

$$23 = 1 + \frac{C}{1}$$

$$22 = C$$

$$y = x^6 + \frac{22}{x^4}$$

y =

$$x^6 + \frac{22}{x^4}$$

35. Find the general solution for the differential equation.

$$\frac{x \frac{dy}{dx} + 5y}{x} = \frac{12x^2}{x}$$

$$\frac{dy}{dx} + \frac{5}{x}y = 12x$$

$$P(x) = \frac{5}{x} \quad Q(x) = 12x$$

$$\begin{aligned} u(x) &= \exp\left[\int P(x) dx\right] \\ &= \exp\left[\int \frac{5}{x} dx\right] \\ &= \exp[5 \ln(x)] \\ &= \exp[\ln(x^5)] \\ &= x^5 \end{aligned}$$

$$y \cdot x^5 = \int 12x(x^5) dx + C$$

$$y \cdot x^5 = \int 12x^6 dx + C$$

$$y \cdot x^5 = \frac{12x^7}{7} + C$$

$$y = \frac{12}{7}x^2 + \frac{C}{x^5}$$

$$y = \frac{12}{7}x^2 + \frac{C}{x^5}$$

36. Find the value of the constant of integration,  $C$ , for the particular solution to the following differential equation.

$$\frac{(x-5)y' + y}{(x-5)} = \frac{x^2 + 14}{(x-5)}, \quad y(3) = 12$$

$$y' + \frac{1}{x-5}y = \frac{x^2 + 14}{x-5}$$

$$P(x) = \frac{1}{x-5} \quad Q(x) = \frac{x^2 + 14}{x-5}$$

$$\begin{aligned} u(x) &= \exp\left[\int \frac{1}{x-5} dx\right] \\ &= \exp[\ln(x-5)] \\ &= x-5 \end{aligned}$$

$$y(x-5) = \int \frac{x^2 + 14}{x-5} (x-5) dx + C$$

$$y(x-5) = \int x^2 + 14 dx + C$$

$$y(x-5) = \frac{x^3}{3} + 14x + C$$

When  $y(3) = 12$

$$12(3-5) = \frac{3^3}{3} + 14(3) + C$$

$$C = -75$$

37. (a) Use summation notation to write the series in compact form.

$$1 - 0.6 + 0.36 - 0.216 + \dots$$

$$\begin{aligned} &= 1 - \frac{6}{10} + \frac{36}{100} - \frac{216}{1000} + \dots \\ &= 1 - \frac{6}{10} + \left(\frac{6}{10}\right)^2 - \left(\frac{6}{10}\right)^3 + \dots \\ &= \sum_{n=0}^{\infty} (-1)^n \left(\frac{6}{10}\right)^n \\ &= \sum_{n=0}^{\infty} \left(\frac{-6}{10}\right)^n \end{aligned}$$

$$\sum_{n=0}^{\infty} \left(\frac{-6}{10}\right)^n$$

Answer: \_\_\_\_\_

(b) Use the sum from (a) and compute the sum.

$$\sum_{n=0}^{\infty} \left(\frac{-6}{10}\right)^n = \frac{1}{1 - (-6/10)} = \frac{1}{1 + 6/10} = \frac{1}{16/10} = \frac{10}{16} = \frac{5}{8}$$

$$5/8$$

Answer: \_\_\_\_\_

38. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$$

Note  $r = 3/2$  and  
 $\left|\frac{3}{2}\right| < 1$  is false  
So the sum diverges

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n = \text{diverges}$$

$$7 + \overbrace{0.3333}^{\text{sum}}$$

39. Express the repeating decimal  $7.\overline{3333}$  as a sum.

$$\begin{aligned} 0.\overline{3333} &= \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \dots \\ &= \frac{3}{10} \left( 1 + \frac{1}{10} + \frac{1}{100} + \dots \right) \\ &= \frac{3}{10} \sum_{n=0}^{\infty} \left( \frac{1}{10} \right)^n \end{aligned}$$

$$7.\overline{3333} = 7 + \frac{3}{10} \sum_{n=0}^{\infty} \left( \frac{1}{10} \right)^n$$

40. If the given series converges, then find its sum. If not, state that it diverges.

$$\begin{aligned} \sum_{n=0}^{\infty} 6 \left( -\frac{1}{9} \right)^n &\rightarrow = \frac{6}{1 - (-1/9)} \\ &= \frac{6}{1 + 1/9} \\ &= \frac{6}{10/9} \\ &= 6 \cdot \frac{9}{10} \\ &= 3 \cdot \frac{9}{5} = \frac{27}{5} \end{aligned}$$

$\frac{27}{5}$

$$\sum_{n=0}^{\infty} 6 \left( -\frac{1}{9} \right)^n = \underline{\hspace{2cm}}$$

41. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{7}{4^n}\right)$$

$$\rightarrow = \sum_{n=0}^{\infty} 7 \left(\frac{1}{4}\right)^n$$

$$= \frac{7}{1 - 1/4}$$

$$= \frac{7}{3/4}$$

$$= 7 \cdot \frac{4}{3} = \frac{28}{3}$$

28/3

$$\sum_{n=0}^{\infty} \left(\frac{7}{4^n}\right) = \underline{\hspace{2cm}}$$

42. Compute

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n}$$

$$\rightarrow = \frac{5^3}{6} + \frac{5^4}{6^2} + \frac{5^5}{6^3} + \dots$$

$$= \frac{5^3}{6} \left(1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \dots\right)$$

$$= \frac{125}{6} \sum_{n=0}^{\infty} \left(\frac{5}{6}\right)^n = \frac{125}{6} \cdot \frac{1}{1 - 5/6}$$

$$= \frac{125}{6} \cdot \frac{1}{1/6} = \frac{125}{6} \cdot \frac{6}{1} = 125$$

125

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n} = \underline{\hspace{2cm}}$$

43. Compute

$$\begin{aligned} & \sum_{n=0}^{\infty} \frac{(-2)^n}{3 \cdot 3^{2n}} \\ &= \sum_{n=0}^{\infty} \frac{(-2)^n}{3 \cdot 3^{2n}} \\ &= \sum_{n=0}^{\infty} \frac{1}{3} \cdot \frac{(-2)^n}{(3^2)^n} \\ &= \sum_{n=0}^{\infty} \frac{1}{3} \left( \frac{-2}{9} \right)^n \\ &= \frac{1/3}{1 - (-2/9)} \\ &= \frac{1/3}{1 + 2/9} \end{aligned}$$

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}}$$

$$\begin{aligned} &= \frac{1/3}{11/9} \\ &= \frac{1}{3} \cdot \frac{9}{11} \\ &= 3/11 \end{aligned}$$

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}} =$$

3/11

44. Evaluate the sum of the following infinite series.

$$\begin{aligned} & \sum_{n=0}^{\infty} (-1)^n \frac{5^{n+1}}{3^{2n}} \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n 5^n \cdot 5^1}{(3^2)^n} \\ &= \sum_{n=0}^{\infty} 5 \left( \frac{-5}{9} \right)^n \\ &= \frac{5}{1 - (-5/9)} \end{aligned}$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{5^{n+1}}{3^{2n}}$$

$$\begin{aligned} &= \frac{5}{14/9} \\ &= \frac{5}{1} \cdot \frac{9}{14} \\ &= \frac{45}{14} \end{aligned}$$

Answer:

45/14

45. Evaluate the sum of the following infinite series.

$$\sum_{n=1}^{\infty} \frac{4(3)^{n-1}}{5^n}$$

$$= \frac{4(3)^0}{5^1} + \frac{4(3)^1}{5^2} + \frac{4(3)^2}{5^3} + \frac{4(3)^3}{5^4} + \dots$$

$$= \frac{4}{5} \left( 1 + \frac{3}{5} + \left(\frac{3}{5}\right)^2 + \left(\frac{3}{5}\right)^3 + \dots \right)$$

$$= \frac{4}{5} \sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^n$$

$$= \frac{4}{5} \cdot \frac{1}{1-3/5}$$

$$= \frac{4}{5} \cdot \frac{1}{2/5} = \frac{4}{5} \cdot \frac{5}{2} = 2$$

2

Answer: \_\_\_\_\_

46. Evaluate the sum of the following infinite series.

$$\sum_{n=1}^{\infty} \left( \frac{3^{n-1}}{4^n} + \frac{(-1)^{n+1}}{9^n} \right)$$

$$= \sum_{n=1}^{\infty} \left( \frac{3^{n-1}}{4^n} + \frac{(-1)^n}{9^n} \right)$$

$$= \sum_{n=1}^{\infty} \left( \frac{1}{3} \left(\frac{3}{4}\right)^n - \left(\frac{-1}{9}\right)^n \right)$$

$$= \frac{1}{3} \left(\frac{3}{4}\right)^1 - \left(\frac{-1}{9}\right)^1$$

$$+ \frac{1}{3} \left(\frac{3}{4}\right)^2 - \left(\frac{-1}{9}\right)^2$$

$$+ \frac{1}{3} \left(\frac{3}{4}\right)^3 - \left(\frac{-1}{9}\right)^3$$

$$+ \dots$$

$$= \frac{1}{3} \left(\frac{3}{4}\right) \left[ 1 + \left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)^2 + \dots \right]$$

$$- \left(\frac{-1}{9}\right) \left[ 1 + \left(\frac{-1}{9}\right) + \left(\frac{-1}{9}\right)^2 + \dots \right]$$

$$= \frac{1}{4} \sum_{n=0}^{\infty} \left(\frac{3}{4}\right)^n + \frac{1}{9} \sum_{n=0}^{\infty} \left(\frac{-1}{9}\right)^n$$

$$= \frac{1}{4} \cdot \frac{1}{1-3/4} + \frac{1}{9} \cdot \frac{1}{1-(-1/9)}$$

1.1

Answer: \_\_\_\_\_

47. Find the radius of convergence for the power series shown below.

$$f(x) = \frac{x}{6x + 13} = \frac{x}{13 + 6x}$$

$$\frac{x}{13} \cdot \frac{1}{1 + 6/13x}$$

$$= \frac{x}{13} \cdot \frac{1}{1 - (-6/13x)}$$

So  $\frac{1}{1 - (-6/13x)} = \sum_{n=0}^{\infty} \left(-\frac{6}{13}x\right)^n$  where  $\left|-\frac{6}{13}x\right| < 1$

Note we only want this

$$|x| < \frac{13}{6}$$

R = 13/6

48. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3(-2x)^n$$

Remember

$$\sum_{n=0}^{\infty} \square^n = \frac{1}{1 - \square} \text{ where } |\square| < 1$$

$$| -2x | < 1$$

$$| 2x | < 1$$

$$2|x| < 1$$

$$|x| < 1/2 = R$$

R = 1/2

49. Find the radius of convergence for the power series shown below.

Remember

$$\sum_{n=0}^{\infty} \square^n = \frac{1}{1-\square} \quad \text{where } |\square| < 1$$

$$|7x^2| < 1$$

$$7|x^2| < 1$$

$$|x^2| < 1/7$$

$$-1/7 < x^2 < 1/7$$

By algebra

$$x^2 < 1/7$$

$$x < \pm \sqrt{1/7}$$

$$|x| < \sqrt{1/7}$$

R =

$\sqrt{1/7}$
--------------

50. Express  $f(x) = \frac{3}{1+2x}$  as a power series and determine its radius of convergence.

$$\frac{3}{1+2x} = \frac{3}{1} \cdot \frac{1}{1+2x} = \frac{3}{1} \cdot \frac{1}{1-(-2x)}$$

$$\frac{1}{1-(-2x)} = \sum_{n=0}^{\infty} (-2x)^n \quad \text{where } |-2x| < 1$$

$$f(x) = \frac{3}{1-(-2x)} = 3 \sum_{n=0}^{\infty} (-2x)^n \quad \text{where } 2|x| < 1$$

$$= \sum_{n=0}^{\infty} 3(-1)^n 2^n x^n \quad \text{where } |x| < 1/2$$

$$\frac{3}{1+2x} =$$

R =

$\sum_{n=0}^{\infty} 3(-1)^n 2^n x^n$
$1/2$

51. Express  $f(x) = \frac{x}{4+3x^2}$  as a power series.

$$\frac{x}{4(1+3x^2/4)} = \frac{x}{4} \cdot \frac{1}{1-(-3x^2/4)}$$

$$\frac{1}{1-(-3x^2/4)} = \sum_{n=0}^{\infty} \left(\frac{-3x^2}{4}\right)^n$$

$$f(x) = \frac{x}{4} \cdot \frac{1}{1-(-3x^2/4)} = \frac{x}{4} \sum_{n=0}^{\infty} \left(\frac{-3x^2}{4}\right)^n$$

$$f(x) = \frac{x}{4} \sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^{2n}}{4^n}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^{2n+1}}{4^{n+1}}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^{2n+1}}{4^{n+1}}$$

$$\frac{x}{4+3x^2} =$$

52. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$\int \sin(x^{3/2}) dx$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

$$\begin{aligned} \sin(x^{3/2}) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (x^{3/2})^{2n+1} \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{3n+3/2} \end{aligned}$$

$$\int \sin(x^{3/2}) dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{3n+3/2} dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \int x^{3n+3/2} dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{x^{3n+5/2}}{3n+5/2}$$

$$= \frac{x^{5/2}}{5/2} - \frac{x^{11/2}}{6 \cdot (3+5/2)} + \frac{x^{17/2}}{5! \cdot (6+5/2)}$$

$$\frac{x^{5/2}}{5/2} - \frac{x^{11/2}}{6 \cdot (3+5/2)} + \frac{x^{17/2}}{5! \cdot (6+5/2)}$$

$$\int \sin(x^{3/2}) dx =$$

53. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\int e^{-3x} dx$$

$$e^{-3x} = \sum_{n=0}^{\infty} \frac{(-3x)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^n}{n!}$$

$$\begin{aligned} \int e^{-3x} dx &= \int \sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{n!} \int x^n dx = \sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{n!} \cdot \frac{x^{n+1}}{(n+1)} \\ &= \frac{(-1)^0 3^0}{0!} \cdot \frac{x^1}{1} + \frac{(-1)^1 3^1}{1!} \cdot \frac{x^2}{2} + \frac{(-1)^2 3^2}{2!} \cdot \frac{x^3}{3} \end{aligned}$$

$$\int e^{-3x} dx =$$

$$x - \frac{3}{2}x^2 + \frac{3}{2}x^3$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

54. Evaluate the following indefinite integral as a series. [Leave your answer as a sum.]

$$\int 5e^{5x^3} dx$$

$$e^{5x^3} = \sum_{n=0}^{\infty} \frac{(5x^3)^n}{n!} = \sum_{n=0}^{\infty} \frac{5^n x^{3n}}{n!}$$

$$5e^{5x^3} = 5 \sum_{n=0}^{\infty} \frac{5^n x^{3n}}{n!} = \sum_{n=0}^{\infty} \frac{5^{n+1}}{n!} x^{3n}$$

$$\int 5e^{5x^3} dx = \int \sum_{n=0}^{\infty} \frac{5^{n+1}}{n!} x^{3n} dx$$

$$= \sum_{n=0}^{\infty} \frac{5^{n+1}}{n!} \int x^{3n} dx$$

$$= \sum_{n=0}^{\infty} \frac{5^{n+1}}{n!} \cdot \frac{x^{3n+1}}{(3n+1)}$$

$$\int 5e^{5x^3} dx =$$

$$\sum_{n=0}^{\infty} \frac{5^{n+1}}{n!} \cdot \frac{x^{3n+1}}{(3n+1)}$$

55. Use the first three terms of the powers series representation of the  $f(x) = \frac{3x}{10+2x}$  to estimate  $f(0.5)$ . Round to 4 decimal places.

$$\frac{3x}{10(1+\frac{2}{10}x)} = \frac{3x}{10} \cdot \frac{1}{1-(-\frac{2}{10}x)}$$

$$\frac{1}{1-(-\frac{2}{10}x)} = \sum_{n=0}^{\infty} \left(-\frac{2}{10}x\right)^n$$

$$f(x) = \frac{3x}{10} \cdot \frac{1}{1-(-\frac{2}{10}x)} = \frac{3x}{10} \sum_{n=0}^{\infty} \left(-\frac{2}{10}x\right)^n$$

$$f(x) = \frac{3x}{10} \sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^n}{10^n}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 2^n \cdot 3 x^{n+1}}{10^{n+1}}$$

$$f(0.5) = \sum_{n=0}^{\infty} \frac{(-1)^n 2^n \cdot 3 \cdot (0.5)^{n+1}}{10^{n+1}}$$

$$= \frac{3(0.5)}{10} - \frac{2 \cdot 3(0.5)^2}{10^2} + \frac{2^2 \cdot 3(0.5)^3}{10^3}$$

$$\approx 0.1365$$

$f(0.5) \approx$  \_\_\_\_\_

0.1365

56. Evaluate the indefinite integral as a power series.

$$\int \frac{x}{3+5x^2} dx$$

$$\frac{x}{3} \cdot \frac{1}{1+5/3x^2} = \frac{x}{3} \cdot \frac{1}{\underbrace{1-(-5/3x^2)}_{\text{sum}}}$$

$$\frac{1}{1-(-5/3x^2)} = \sum_{n=0}^{\infty} \left(-\frac{5}{3}x^2\right)^n$$

$$\begin{aligned} \frac{x}{3} \cdot \frac{1}{1-(-5/3x^2)} &= \frac{x}{3} \sum_{n=0}^{\infty} \left(-\frac{5}{3}x^2\right)^n \\ &= \frac{x}{3} \sum_{n=0}^{\infty} \frac{(-1)^n 5^n x^{2n}}{3^n} \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n 5^n x^{2n+1}}{3^{n+1}} \end{aligned}$$

$$\begin{aligned} \int \frac{x}{3+5x^2} dx &= \sum_{n=0}^{\infty} \frac{(-1)^n 5^n}{3^{n+1}} \cdot \int x^{2n+1} dx \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n 5^n}{3^{n+1}} \cdot \frac{x^{2n+2}}{2n+2} \end{aligned}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n 5^n}{3^{n+1}} \cdot \frac{x^{2n+2}}{2n+2}$$

Answer: \_\_\_\_\_

57. Use a power series to approximate the definite integral using the first 3 terms of the series.

$$\int_0^{0.24} \frac{x}{5+x^6} dx$$

$$\frac{x}{5+x^6} = \frac{x}{5-(-x^6)} = \frac{x}{5[1-(-x^6/5)]} = \frac{x}{5} \cdot \frac{1}{1-(-x^6/5)}$$

$$\frac{1}{1-(-x^6/5)} = \sum_{n=0}^{\infty} \left(\frac{-x^6}{5}\right)^n = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{5^n}$$

$$\frac{x}{5} \cdot \frac{1}{1-(-x^6/5)} = \frac{x}{5} \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{5^n} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+1}}{5^{n+1}}$$

$$\int_0^{0.24} \frac{x}{5+x^6} dx = \int_0^{0.24} \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+1}}{5^{n+1}} dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{5^{n+1}} \int_0^{0.24} x^{6n+1} dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{5^{n+1}} \cdot \left[ \frac{x^{6n+2}}{6n+2} \right]_0^{0.24}$$

$$= \left( \frac{1}{5} \cdot \frac{x^2}{2} - \frac{1}{5^2} \cdot \frac{x^8}{8} + \frac{1}{5^3} \cdot \frac{x^{14}}{14} \right) \Big|_0^{0.24}$$

$$\approx 0.00576$$

$$\int_0^{0.24} \frac{x}{5+x^6} dx \approx$$

0.00576

58. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.11} \frac{1}{1+x^4} dx$$

$$\frac{1}{1+x^4} = \frac{1}{1-(-x^4)} = \sum_{n=0}^{\infty} (-x^4)^n = \sum_{n=0}^{\infty} (-1)^n x^{4n}$$

$$\begin{aligned} \int_0^{0.11} \frac{1}{1+x^4} dx &= \int_0^{0.11} \sum_{n=0}^{\infty} (-1)^n x^{4n} dx \\ &= \sum_{n=0}^{\infty} (-1)^n \int_0^{0.11} x^{4n} dx \\ &= \sum_{n=0}^{\infty} (-1)^n \left[ \frac{x^{4n+1}}{4n+1} \right]_0^{0.11} \\ &= \left( x - \frac{x^5}{5} + \frac{x^9}{9} - \frac{x^{13}}{13} \right) \Big|_0^{0.11} \end{aligned}$$

$$\int_0^{0.11} \frac{1}{1+x^4} dx \approx$$

0.11000

59. Use a power series to approximate the definite integral using the first 3 terms of the series. Round to 5 decimal places.

$$\int_0^{0.23} e^{-x^2} dx$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n}$$

$$\begin{aligned} \int_0^{0.23} e^{-x^2} dx &= \int_0^{0.23} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n} dx \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int_0^{0.23} x^{2n} dx \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left[ \frac{x^{2n+1}}{2n+1} \right]_0^{0.23} \\ &= \left( \frac{x}{1} - \frac{x^3}{1!(3)} + \frac{x^5}{2!(5)} \right) \Big|_0^{0.23} \end{aligned}$$

$$= \left( x - \frac{x^3}{3} + \frac{x^5}{10} \right) \Big|_0^{0.23}$$

$$\int_0^{0.23} e^{-x^2} dx \approx$$

0.226

60. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.45} 4x \cos(\sqrt{x}) dx$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

$$\begin{aligned} \cos(\sqrt{x}) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x^{1/2})^{2n} \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^n \end{aligned}$$

$$\begin{aligned} f(x) = 4x \cos(\sqrt{x}) &= 4x \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^n \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4x^{n+1} \end{aligned}$$

$$\begin{aligned} \int_0^{0.45} 4x \cos(\sqrt{x}) dx &= \int_0^{0.45} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4x^{n+1} dx \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4 \int_0^{0.45} x^{n+1} dx \end{aligned}$$

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4 \frac{x^{n+2}}{n+2} \Big|_0^{0.45} \\ &= \left( \frac{4x^2}{0!(2)} - \frac{4x^3}{2!(3)} + \frac{4x^4}{4!(4)} - \frac{4x^5}{6!(5)} \right) \Big|_0^{0.45} \\ &= \left( 2x^2 - \frac{2x^3}{3} + \frac{x^4}{24} - \frac{x^5}{900} \right) \Big|_0^{0.45} \end{aligned}$$

0.34593

61. Use the first five terms of the Macluarin series for  $f(x) = \ln(1+x)$  to evaluate  $\ln(1.44)$ . Round to 5 decimal places.

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$$

Note  $1.56 = 1 + 0.56$

$$\ln(1+0.56) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (0.56)^n = 0.56 - \frac{(0.56)^2}{2} + \frac{(0.56)^3}{3}$$

0.46174

$$\ln(1.44) \approx \underline{\hspace{2cm}}$$

62. Use the first 4 terms of the Macluarin series for  $f(x) = \sin(x)$  to evaluate  $\sin(0.75)$ . Round to 5 decimal places.

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\sin(0.75) = \sum_{n=0}^{\infty} \frac{(-1)^n (0.75)^{2n+1}}{(2n+1)!} = \frac{0.75}{1!} - \frac{(0.75)^3}{3!} + \frac{(0.75)^5}{5!} - \frac{(0.75)^7}{7!}$$

0.74631

$\sin(0.75) \approx$  \_\_\_\_\_

63. Given  $f(x, y) = 3x^3y^2 - x^2y^{1/3}$ , evaluate  $f(3, -8)$ .

$$f(3, -8) = 3(3)^3(-8)^2 - (3)^2(-8)^{1/3}$$

5202

$f(3, -8) =$  \_\_\_\_\_

64. Find the domain of

$$f(x, y) = \frac{-5x}{\sqrt{x+9y+1}}$$

$$\frac{1}{\sqrt{?}} \rightarrow ? > 0$$

$$x + 9y + 1 > 0$$

$$\{(x, y) \mid x + 9y + 1 > 0\}$$

Domain = \_\_\_\_\_

65. Find the domain of

$$f(x, y) = \frac{\sqrt{x+y-1}}{\ln(y-11)-9}$$

$$\sqrt{?} \rightarrow ? \geq 0$$

$$\sqrt{x+y-1} \rightarrow x+y-1 \geq 0$$

$$x+y \geq 1$$

$$\ln(?) \rightarrow ? > 0$$

$$\ln(y-11) \rightarrow y-11 > 0$$

$$y > 11$$

$$\frac{1}{?} \rightarrow ? \neq 0$$

$$\ln(y-11)-9 \neq 0$$

$$\ln(y-11) \neq 9$$

$$y-11 \neq e^9$$

$$y \neq e^9 + 11$$

$$\text{Domain} = \{(x, y) \mid x+y \geq 1, y > 11, y \neq 11+e^9\}$$

66. Find the domain of

$$f(x, y) = \frac{\ln(x^2 - y + 3)}{\sqrt{x-6}}$$

$$\ln(?) \rightarrow ? > 0$$

$$\ln(x^2 - y + 3) \rightarrow x^2 - y + 3 > 0$$

$$x^2 + 3 > y$$

$$\frac{1}{\sqrt{?}} \rightarrow ? > 0$$

$$\frac{1}{\sqrt{x-6}} \rightarrow x-6 > 0$$

$$x > 6$$

$$\text{Domain} = \{(x, y) \mid x > 6, x^2 + 3 > y\}$$

67. Describe the indicated level curves  $f(x, y) = C$

$$f(x, y) = \ln(x^2 + y^2) \quad C = \ln(36)$$

- (a) Parabola with vertices at  $(0, 0)$
- (b) Circle with center at  $(0, \ln(36))$  and radius 6
- (c) Parabola with vertices at  $(0, \ln(36))$
- (d) Circle with center at  $(0, 0)$  and radius 6
- (e) Increasing Logarithm Function

$$\begin{aligned}\ln(x^2 + y^2) &= \ln(36) \\ x^2 + y^2 &= 36 \\ x^2 + y^2 &= 6^2\end{aligned}$$

68. What do the level curves for the following function look like?

$$f(x, y) = \ln(y - e^{5x})$$

- (a) Increasing exponential functions
- (b) Rational Functions with x-axis symmetry
- (c) Natural logarithm functions
- (d) Decreasing exponential functions
- (e) Rational Functions with y-axis symmetry

$$\begin{aligned}\ln(y - e^{5x}) &= C \\ y - e^{5x} &= e^C \\ y - e^{5x} &= C \\ y &= e^{5x} + C\end{aligned}$$

69. What do the level curves for the following function look like?

$$f(x, y) = \sqrt{y + 4x^2}$$

- (a) Lines
- (b) Parabolas
- (c) Circles
- (d) Point at the origin
- (e) Ellipses
- (f) Hyperbolas

$$\begin{aligned}\sqrt{x^2 + y^2} &= C \\ x^2 + y^2 &= C^2\end{aligned}$$

70. What do the level curves for the following function look like?

$$f(x, y) = \cos(y + 4x^2)$$

- (a) Lines
- (b) Parabolas
- (c) Circles
- (d) Point at the origin
- (e) Ellipses
- (f) Hyperbolas

$$\begin{aligned}\cos(y + 4x^2) &= C \\ y + 4x^2 &= \cos^{-1}(C) \\ y + 4x^2 &= C \\ y &= -4x^2 + C\end{aligned}$$

71. What do the level curves for the following function look like?

$$f(x, y) = \ln(8y - 5x^2)$$

- (a) logarithmic curves
- (b) lines
- (c) a point at the origin
- (d) circles
- (e) hyperbolas
- (f) parabolas

$$\begin{aligned}\ln(8y - 5x^2) &= C \\ 8y - 5x^2 &= e^C = C \\ 8y &= C + 5x^2\end{aligned}$$

72. For the following function  $f(x, y)$ , evaluate  $f_y(-2, -3)$ .

$$f(x, y) = 8x^4y^5 + 3x^3 - 12y^2$$

$$\begin{aligned}f_y(x, y) &= \frac{d}{dy} (8x^4y^5 + 3x^3 - 12y^2) \\ &= 8x^4 \frac{d}{dy} (y^5) + 3x^3 \frac{d}{dy} (1) - \frac{d}{dy} (12y^2) \\ &= (8x^4)(5y^4) + (3x^3)(0) - 24y \\ &= 40x^4y^4 - 24y\end{aligned}$$

$$\begin{aligned}f_y(-2, -3) &= 40(-2)^4(-3)^4 - 24(-3) \\ &= 51912\end{aligned}$$

$$f_y(-2, -3) =$$

51912

73. Compute  $f_x(6, 5)$  when

$$f(x, y) = \frac{(6x - 6y)^2}{\sqrt{y^2 - 1}}$$

$$\begin{aligned} f_x(x, y) &= \frac{d}{dx} \left( \frac{(6x - 6y)^2}{\sqrt{y^2 - 1}} \right) \\ &= \frac{1}{\sqrt{y^2 - 1}} \frac{d}{dx} ((6x - 6y)^2) \\ &= \frac{1}{\sqrt{y^2 - 1}} \cdot 2(6x - 6y) \frac{d}{dx} (6x + 6y) \\ &= \frac{1}{\sqrt{y^2 - 1}} \cdot 2(6x - 6y) \cdot 6 \\ &= \frac{72x - 72y}{\sqrt{y^2 - 1}} \end{aligned}$$

$$f_x(6, 5) =$$

$$\frac{72}{\sqrt{24}}$$

74. Given the function  $f(x, y) = \sin(x^2 y)$ , evaluate  $f_x\left(\frac{1}{2}, \pi\right)$ .

$$f_x = \cos(x^2 y) \cdot 2xy$$

$$\begin{aligned} f_x\left(\frac{1}{2}, \pi\right) &= \cos\left(\frac{1}{4} \cdot \pi\right) \cdot 2 \cdot \frac{1}{2} \cdot \pi \\ &= \frac{\sqrt{2}}{2} \cdot \pi \end{aligned}$$

$$f_x\left(\frac{1}{2}, \pi\right) =$$

$$\frac{\sqrt{2}}{2} \pi$$

75. Find the first order partial derivatives of

$$f(x, y) = 3x^2 \cdot \frac{y^3}{(y-1)^2} \quad f(x, y) = \frac{3x^2 y^3}{(y-1)^2}$$

$$f_x(x, y) = \frac{d}{dx} \left( 3x^2 \cdot \frac{y^3}{(y-1)^2} \right) = \frac{y^3}{(y-1)^2} \cdot \frac{d}{dx} (3x^2) = \frac{y^3}{(y-1)^2} \cdot 6x$$

$$f_y(x, y) = \frac{d}{dy} \left( 3x^2 \cdot \frac{y^3}{(y-1)^2} \right) = 3x^2 \frac{d}{dy} \left( \frac{y^3}{(y-1)^2} \right) = 3x^2 \left( \frac{3y^2(y-1)^2 - y^3 \cdot 2(y-1)}{(y-1)^4} \right)$$

$$= 3x^2 \left( \frac{\cancel{(y-1)} [3y^2(y-1) - 2y^3]}{(y-1)^{\cancel{4}3}} \right) = \frac{3x^2 (3y^3 - 3y^2 - 2y^3)}{(y-1)^3}$$

$$= \frac{3x^2 (y^3 - 3y^2)}{(y-1)^3}$$

$f_x(x, y) =$	$\frac{6xy^3}{(y-1)^2}$
$f_y(x, y) =$	$\frac{3x^2(y^3 - 3y^2)}{(y-1)^3}$

76. Find the first order partial derivatives of

$$f(x, y) = x \sin(xy)$$

$$f_x(x, y) = \frac{d}{dx} (x \sin(xy)) = \frac{d}{dx} (x) \sin(xy) + x \frac{d}{dx} (\sin(xy))$$

$$= \sin(xy) + x \cos(xy) \frac{d}{dx} (xy)$$

$$= \sin(xy) + x \cdot y \cos(xy)$$

$$f_y(x, y) = \frac{d}{dy} (x \sin(xy)) = x \frac{d}{dy} (\sin(xy))$$

$$= x \cos(xy) \frac{d}{dy} (xy)$$

$$= x^2 \cos(xy)$$

$f_x(x, y) =$	$\sin(xy) + xy \cos(xy)$
$f_y(x, y) =$	$x^2 \cos(xy)$

77. Find the first order partial derivatives of  $f(x, y) = (xy - 1)^2$

$$\begin{aligned}f_x(x, y) &= \frac{d}{dx} \left( (xy - 1)^2 \right) = 2(xy - 1) \frac{d}{dx} (xy - 1) \\ &= 2(xy - 1) y \\ &= 2xy^2 - 2y\end{aligned}$$

$$\begin{aligned}f_y(x, y) &= \frac{d}{dy} \left( (xy - 1)^2 \right) = 2(xy - 1) \frac{d}{dy} (xy - 1) \\ &= 2(xy - 1) x \\ &= 2x^2y - 2x\end{aligned}$$

$f_x(x, y) =$

$$2xy^2 - 2y$$

$f_y(x, y) =$

$$2x^2y - 2x$$

78. Find the first order partial derivatives of  $f(x, y) = xe^{x^2+xy+y^2}$

$$\begin{aligned}f_x(x, y) &= \frac{d}{dx} (x) e^{x^2+xy+y^2} + x \frac{d}{dx} (e^{x^2+xy+y^2}) \\ &= e^{x^2+xy+y^2} + x(e^{x^2+xy+y^2})(2x+y) \\ &= (1+2x^2+xy)e^{x^2+xy+y^2}\end{aligned}$$

$$\begin{aligned}f_y(x, y) &= x \frac{d}{dy} (e^{x^2+xy+y^2}) = x(e^{x^2+xy+y^2})(x+2y) \\ &= (x^2+2xy)e^{x^2+xy+y^2}\end{aligned}$$

$f_x(x, y) =$

$$(1+2x^2+xy)e^{x^2+xy+y^2}$$

$f_y(x, y) =$

$$(x^2+2xy)e^{x^2+xy+y^2}$$

79. Find the first order partial derivatives of  $f(x, y) = -7 \tan(x^7 y^8)$

$$f_x(x, y) = -7 \frac{d}{dx} (\tan(x^7 y^8)) = -7 \sec^2(x^7 y^8) \frac{d}{dx} (x^7 y^8)$$

$$= -7 \cdot 7 x^6 y^8 \sec^2(x^7 y^8) = -49 x^6 y^8 \sec^2(x^7 y^8)$$

$$f_y(x, y) = -7 \frac{d}{dy} (\tan(x^7 y^8)) = -7 \sec^2(x^7 y^8) \frac{d}{dy} (x^7 y^8)$$

$$= -7 \cdot 8 x^7 y^7 \sec^2(x^7 y^8)$$

$$= -56 x^7 y^7 \sec^2(x^7 y^8)$$

$f_x(x, y) =$	$-49 x^6 y^8 \sec^2(x^7 y^8)$
$f_y(x, y) =$	$-56 x^7 y^7 \sec^2(x^7 y^8)$

80. Find the first order partial derivatives of  $f(x, y) = y \cos(x^2 y)$

$$f_x(x, y) = y \frac{d}{dx} (\cos(x^2 y)) = y (-\sin(x^2 y)) \frac{d}{dx} (x^2 y) = -y \sin(x^2 y) [2xy]$$

$$= -2x y^2 \sin(x^2 y)$$

$$f_y(x, y) = \frac{d}{dy} (y) \cos(x^2 y) + y \frac{d}{dy} (\cos(x^2 y))$$

$$= \cos(x^2 y) + y (-\sin(x^2 y)) \frac{d}{dy} (x^2 y)$$

$$= \cos(x^2 y) - y \sin(x^2 y) [x^2]$$

$$= \cos(x^2 y) - x^2 y \sin(x^2 y)$$

$f_x(x, y) =$	$-2x y^2 \sin(x^2 y)$
$f_y(x, y) =$	$\cos(x^2 y) - x^2 y \sin(x^2 y)$

81. Find the first order partial derivatives of  $f(x, y) = xe^{xy}$

$$\begin{aligned}f_x &= \frac{\partial}{\partial x} (xe^{xy}) = \frac{\partial}{\partial x} (x) e^{xy} + x \frac{\partial}{\partial x} (e^{xy}) \\&= e^{xy} + xe^{xy} \frac{\partial}{\partial x} (xy) \\&= e^{xy} + xe^{xy} (y) \\&= e^{xy} (1 + xy)\end{aligned}$$

$$\begin{aligned}f_y &= \frac{\partial}{\partial y} (xe^{xy}) = x \frac{\partial}{\partial y} (e^{xy}) \\&= xe^{xy} \frac{\partial}{\partial y} (xy) \\&= xe^{xy} \cdot x \\&= x^2 e^{xy}\end{aligned}$$

$f_x(x, y) =$  \_\_\_\_\_

$f_y(x, y) =$  \_\_\_\_\_

$e^{xy} (1 + xy)$
$x^2 e^{xy}$