

Please show **all** your work! Answers without supporting work will not be given credit.
Write answers in spaces provided.

Solutions

Name: _____

1. Evaluate the indefinite integral.

$$\int 18x \cos(x^2) dx$$

$$\begin{aligned} \frac{u=x^2}{du=2x dx} & \int 18x \cos(u) \frac{du}{2x} = \int 9 \cos(u) du = 9 \sin(u) + C \\ \frac{du}{2x} = dx & = 9 \sin(x^2) + C \end{aligned}$$

$$\int 18x \cos(x^2) dx = \underline{9 \sin(x^2) + C}$$

2. Evaluate the indefinite integral.

$$\int 11x^2 e^{-4x^3} dx$$

$$\begin{aligned} \frac{u=-4x^3}{du=-12x^2 dx} & \int 11x^2 e^u \frac{du}{-12x^2} = -\frac{11}{12} \int e^u du = -\frac{11}{12} e^u + C \\ \frac{du}{-12x^2} = dx & = -\frac{11}{12} e^{-4x^3} + C \end{aligned}$$

$$\int 11x^2 e^{-4x^3} dx = \underline{-\frac{11}{12} e^{-4x^3} + C}$$

3. A forestry company estimates that acres of forest available for logging will increase at a rate given by $R'(t) = \frac{56}{\sqrt{t+7}}$ for $0 \leq t \leq 20$ where $R'(t)$ is the rate of new acreage becoming available in thousands of acres per year, t years after the current year. How many acres of forest will become available for logging over the first 5 years? Round your answer to the nearest thousand acres.

$$\int_0^5 56(t+7)^{-1/2} dt \quad \frac{u=t+7}{du=dt} \quad \int 56u^{-1/2} du = 56u^{1/2} \cdot \frac{2}{1}$$

$$= 112(t+7)^{1/2} \Big|_0^5$$

$$\approx 92$$

Answer: 92

4. Find the area under the curve $y = 7 \cos(4x)$ for $0 \leq x \leq \pi/2$.

$$\int_0^{\pi/2} 7 \cos(4x) dx \quad \frac{u=4x}{du=4dx} \quad \int 7 \cos(u) \frac{du}{4} = \frac{7}{4} \sin(u)$$

$$\frac{du}{4} = dx$$

$$= \frac{7}{4} \sin(4x) \Big|_0^{\pi/2} = \frac{7}{4} \sin\left(\frac{4\pi}{2}\right) - \frac{7}{4} \sin(4 \cdot 0)$$

$$= 0 - 0$$

Area = 0

5. Evaluate the indefinite integral

$$\int \frac{\ln(7x)}{x} dx$$

$$\begin{aligned} \frac{u = \ln(7x)}{du = \frac{1}{7x} \cdot 7 dx} & \quad \int \frac{u}{x} \cdot x du = \int u du = \frac{u^2}{2} + C \\ du = \frac{1}{7x} \cdot 7 dx & \\ du = \frac{1}{x} dx & \\ x du = dx & \quad = \frac{(\ln(7x))^2}{2} + C \end{aligned}$$

$$\int \frac{\ln(7x)}{x} dx = \frac{(\ln(7x))^2}{2} + C$$

6. Evaluate the definite integral.

$$\int_1^e \frac{\ln(x)}{x} dx$$

$$\begin{aligned} \frac{u = \ln(x)}{du = \frac{1}{x} dx} & \quad \int \frac{u}{x} \cdot x du = \int u du = \frac{u^2}{2} \\ du = \frac{1}{x} dx & \\ x du = dx & \quad = \left. \frac{(\ln(x))^2}{2} \right]_1^e \\ & \quad = \frac{(\ln(e))^2}{2} - \frac{(\ln(1))^2}{2} \end{aligned}$$

$$\int_1^e \frac{\ln(x)}{x} dx = \frac{1}{2}$$

7. Evaluate the indefinite integral.

$$\int x^3 \ln(9x) dx$$
$$\frac{u = \ln(9x)}{du = \frac{1}{9x} \cdot 9 dx} \quad \frac{dv = x^3 dx}{v = \frac{x^4}{4} dx} \quad uv - \int v du = \frac{x^4 \ln(9x)}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x} dx$$
$$du = \frac{1}{x} dx \quad v = \frac{x^4}{4} dx$$

$$= \frac{x^4 \ln(9x)}{4} - \int \frac{1}{4} x^3 dx = \frac{x^4 \ln(9x)}{4} - \frac{1}{4} \cdot \frac{x^4}{4} + C$$

$$\int x^3 \ln(9x) dx = \frac{x^4 \ln(9x)}{4} - \frac{x^4}{16} + C$$

8. Evaluate

$$\int 3x \ln(x^6) dx$$
$$= \int 3x \cdot 6 \ln(x) dx = \int 18x \ln(x) dx$$
$$\frac{u = \ln(x)}{du = \frac{1}{x} dx} \quad \frac{dv = 18x dx}{v = \frac{18x^2}{2}} \quad uv - \int v du = 9x^2 \ln(x) - \int 9x^2 \cdot \frac{1}{x} dx$$
$$v = 9x^2$$
$$= 9x^2 \ln(x) - \int 9x dx = 9x^2 \ln(x) - \frac{9x^2}{2} + C$$

$$\int 3x \ln(x^6) dx = \frac{9x^2 \ln(x) - \frac{9x^2}{2}}{2} + C$$

9. Evaluate the indefinite integral.

$$\begin{aligned} & \int 20x \sin(2x) dx \\ \frac{u=20x}{du=20dx} \quad \frac{dv=\sin(2x)dx}{v=-\frac{\cos(2x)}{2}} & \quad uv - \int v du \\ &= 20x \left(-\frac{\cos(2x)}{2} \right) - \int \left(-\frac{\cos(2x)}{2} \right) 20 dx \\ &= -10x \cos(2x) + \int 10 \cos(2x) dx \\ &= -10x \cos(2x) + \frac{10 \sin(2x)}{2} + C \end{aligned}$$

$$\int 20x \sin(2x) dx = \underline{-10x \cos(2x) + 5 \sin(2x) + C}$$

10. Evaluate the indefinite integral.

$$\begin{aligned} & \int 18x \cos(3x) dx \\ \frac{u=18x}{du=18dx} \quad \frac{dv=\cos(3x)dx}{v=\frac{\sin(3x)}{3}} & \quad uv - \int v du = 18x \left(\frac{\sin(3x)}{3} \right) - \int \left(\frac{\sin(3x)}{3} \right) 18 dx \\ &= 6x \sin(3x) - \int 6 \sin(3x) dx \\ &= 6x \sin(3x) + \frac{6 \cos(3x)}{3} + C \end{aligned}$$

$$\int 18x \cos(3x) dx = \underline{6x \sin(3x) + 2 \cos(3x) + C}$$

11. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{7x - 5}{x^2(x^2 + 9)}$$

(A) $\frac{A}{x} + \frac{B}{x} + \frac{Cx + D}{x^2 + 9}$

(B) $\frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + 9}$

(C) $\frac{A}{x} + \frac{Bx + C}{x^2} + \frac{Dx + E}{x^2 + 9}$

(D) $\frac{Ax + B}{x^2} + \frac{Cx + D}{x^2 + 9}$

(E) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x + 3} + \frac{D}{x - 3}$

(F) $\frac{Ax + B}{x^2} + \frac{C}{x + 3} + \frac{D}{x - 3}$

12. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{2x}{(x - 1)^2(x^2 + 5)}$$

(A) $\frac{A}{x - 1} + \frac{Bx + C}{(x - 1)^2} + \frac{D}{x^2 + 5}$

(B) $\frac{A}{(x - 1)^2} + \frac{Bx + C}{x^2 + 5}$

(C) $\frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{Cx + D}{x^2 + 5}$

(D) $\frac{A}{x - 1} + \frac{Bx + C}{(x - 1)^2} + \frac{Dx + E}{x^2 + 5}$

(E) $\frac{A}{x - 1} + \frac{Bx + C}{x^2 + 5}$

(F) $\frac{Ax + B}{(x - 1)^2} + \frac{Cx + D}{x^2 + 5}$

13. Evaluate the improper integral or state that it is divergent.

$$\begin{aligned} \lim_{N \rightarrow \infty} \int_3^N 4x^{-5} dx &= \lim_{N \rightarrow \infty} \left[\frac{4x^{-4}}{-4} \right]_3^N = \lim_{N \rightarrow \infty} \left[-\frac{1}{x^4} \right]_3^N \\ &= \lim_{N \rightarrow \infty} \left(-\frac{1}{N^4} - \left(-\frac{1}{3^4} \right) \right) = \frac{1}{3^4} = \frac{1}{81} \end{aligned}$$

$$\int_3^{\infty} \frac{4}{x^5} dx = \underline{\underline{1/81}}$$

14. Evaluate the following integral;

$$\begin{aligned} \lim_{N \rightarrow \infty} \int_0^N e^{-x/6} dx &\quad \begin{array}{l} u = -x/6 \\ du = -\frac{1}{6} dx \\ -6 du = dx \end{array} \quad \lim_{N \rightarrow \infty} \int -6e^u du \\ &= \lim_{N \rightarrow \infty} (-6e^u) = \lim_{N \rightarrow \infty} \left(-6e^{-x/6} \right) \Big|_0^N = \lim_{N \rightarrow \infty} \left(-6e^{-N/6} - (-6e^0) \right) \\ &= 6 \end{aligned}$$

$$\int_0^{\infty} e^{-x/6} dx = \underline{\underline{6}}$$

15. Evaluate the definite integral.

$$\int_2^{\infty} \frac{dx}{3x+1}$$
$$= \lim_{N \rightarrow \infty} \int_2^N \frac{dx}{3x+1} \quad \begin{array}{l} u=3x+1 \\ du=3dx \\ \frac{du}{3}=dx \end{array} \quad \lim_{N \rightarrow \infty} \int \frac{1}{u} \frac{du}{3} = \lim_{N \rightarrow \infty} \frac{1}{3} \ln|u|$$
$$= \lim_{N \rightarrow \infty} \left. \frac{1}{3} \ln|3x+1| \right]_2^N = \lim_{N \rightarrow \infty} \left(\frac{1}{3} \ln|3 \cdot N + 1| - \frac{1}{3} \ln|3 \cdot 2 + 1| \right)$$
$$= \infty - \frac{1}{3} \ln(7)$$

$$\int_2^{\infty} \frac{dx}{3x+1} = \underline{\infty \Rightarrow \text{diverges}}$$

16. Set up the integral that represents the **AREA** of the region bounded by the following curves:

$$y = \frac{1}{2}x^2, \quad y = -x^2 + 6$$

Bounds: $\frac{1}{2}x^2 = -x^2 + 6$

$$\frac{3}{2}x^2 = 6$$

$$x^2 = 6 \cdot \frac{2}{3}$$

$$x^2 = 4$$

$$x = \pm 2$$

Test Pt: $x=0$

$$y = \frac{1}{2}x^2 \rightarrow y=0 \rightarrow \text{Bottom}$$

$$y = -x^2 + 6 \rightarrow y=6 \rightarrow \text{Top}$$

$$\text{Area} = \int_{-2}^2 (-x^2 + 6 - \frac{1}{2}x^2) dx$$

17. Find the **AREA** of the region bounded by $y = 2x - x^2$ and $y = x^2$.

Bounds: $2x - x^2 = x^2$
 $2x - 2x^2 = 0$
 $2x(1-x) = 0$
 $x = 0, 1$

Test Pt: $x = \frac{1}{2}$

$y = 2x - x^2 \rightarrow y = \frac{3}{4} \rightarrow \text{Top}$
 $y = x^2 \rightarrow y = \frac{1}{4} \rightarrow \text{Bottom}$

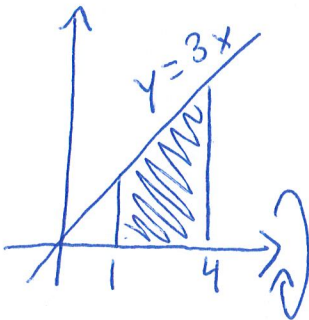
$$\begin{aligned} A &= \int_0^1 (2x - x^2 - x^2) dx \\ &= \int_0^1 (2x - 2x^2) dx \\ &= \left(\frac{2x^2}{2} - \frac{2x^3}{3} \right) \Big|_0^1 \\ &= \left(x^2 - \frac{2x^3}{3} \right) \Big|_0^1 = \frac{1}{3} \end{aligned}$$

Area = $\frac{1}{3}$

18. Find the **VOLUME** of the solid that results by revolving the region enclosed by the following curves about the x-axis.

and

$y = 3x, x = 1, x = 4, \text{ and } y = 0$



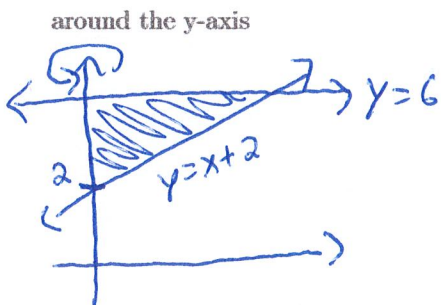
Disk Method $\Rightarrow dx$

$$\begin{aligned} V &= \pi \int_1^4 (3x)^2 dx = \pi \int_1^4 9x^2 dx \\ &= \pi \left[\frac{9x^3}{3} \right]_1^4 = 3\pi \left[x^3 \right]_1^4 \\ &= 3\pi (4^3 - 1^3) = 189\pi \end{aligned}$$

Volume = 189π

19. Find the **VOLUME** of the solid generated by rotating the region bounded by

$$y = x + 2, \quad x = 0, \quad y = 6$$



y-axis + Disk $\Rightarrow dy$

$$\Rightarrow y = x + 2 \leftrightarrow x = y - 2$$

$$V = \pi \int_2^6 (y-2)^2 dy \quad \frac{u=y-2}{du=dy} \quad \pi \int u^3 du$$

$$= \pi \frac{u^4}{4} = \frac{\pi}{4} (y-2)^2 \Big]_2^6$$

$$= \frac{\pi}{4} \left((6-2)^2 - (2-2)^2 \right) = 4\pi$$

Volume = _____

20. **SET-UP** using the washer method. the **VOLUME** of the region bounded by

$$y = x^2, \quad y = 2x$$

around the x-axis

(A) $\pi \int_0^2 (2x - x^2)^2 dx$

(B) $\pi \int_0^2 (4x^2 - x^4) dx$

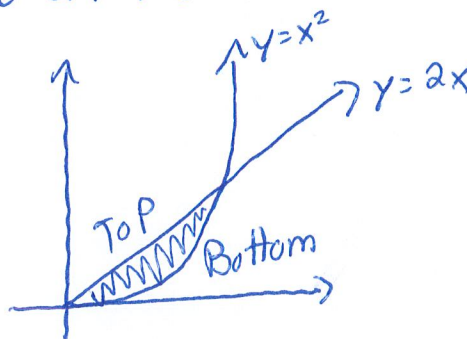
(C) $\pi \int_0^2 (2x - x^2) dx$

(D) $\pi \int_0^2 (x^2 - 2x) dx$

(E) $\pi \int_0^2 (x^4 - 4x^2) dx$

(F) $2\pi \int_0^2 (x^3 - 2x^2) dx$

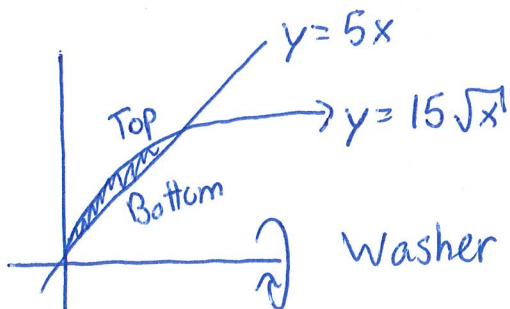
Note all the bounds are the same.



$$\pi \int_0^2 (2x)^2 - (x^2)^2 dx$$

21. Set-up the definite integral that would calculate the **VOLUME** of the region bounded by the following curves when rotated about the x -axis.

$$y = 5x \text{ and } y = 15\sqrt{x}$$



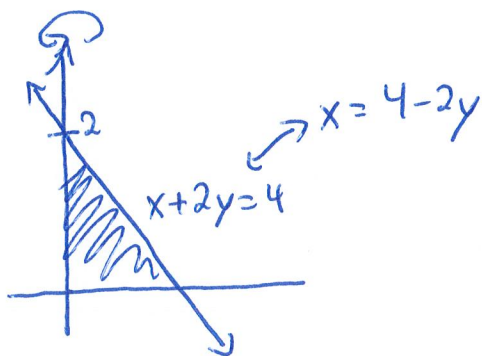
Bounds: $5x = 15\sqrt{x}$
 $(5x)^2 = (15\sqrt{x})^2$
 $25x^2 = 225x$
 $25x^2 - 225x = 0$
 $25x(x - 9) = 0$
 $x = 0, 9$

$$V = \pi \int (15\sqrt{x})^2 - (5x)^2 dx$$

$$\text{VOLUME} = \pi \int_0^9 (225x - 25x^2) dx$$

22. Find the **VOLUME** of the solid obtained by revolving the region bounded by the following curves about the y -axis.

$$x + 2y = 4, x = 0, \text{ and } y = 0$$



$$\begin{aligned} & \left. \begin{aligned} 2y &= 4 - x \\ y &= 2 - \frac{x}{2} \end{aligned} \right\} \begin{aligned} u &= 2y - 4 & \pi \int u^2 \cdot \frac{du}{2} \\ du &= 2dy \\ \frac{du}{2} &= dy \end{aligned} \\ & = \frac{\pi}{2} \frac{u^3}{3} = \frac{\pi}{6} (2y - 4)^3 \Big|_0^2 \\ & = \frac{\pi}{6} ((2 \cdot 2 - 4)^3 - (0 - 4)^3) \end{aligned}$$

Disk + y -axis $\Rightarrow dy$

$$\begin{aligned} V &= \pi \int_0^2 (4 - 2y)^2 dy \\ &= \pi \int_0^2 (2y - 4)^2 dy \end{aligned}$$

$$\text{VOLUME} = \frac{32\pi}{3}$$

23. Find the general solution of the given differential equation.

$$\frac{dy}{dx} = \frac{4x}{y}$$

$$\begin{aligned}y dy &= 4x dx \\ \int y dy &= \int 4x dx \\ \frac{y^2}{2} &= \frac{4x^2}{2} + C \\ y^2 &= 4x^2 + 2C \\ y^2 &= 4x^2 + C \\ y &= \pm \sqrt{4x^2 + C}\end{aligned}$$

$$y = \pm \sqrt{4x^2 + C}$$

24. Find the particular solution to the given differential equation if $y(2) = 3$

$$\frac{dy}{dx} = \frac{x}{y^2}$$

$$\begin{aligned}y^2 dy &= x dx \\ \int y^2 dy &= \int x dx \\ \frac{y^3}{3} &= \frac{x^2}{2} + C \\ y^3 &= \frac{3x^2}{2} + 3C \\ y^3 &= \frac{3x^2}{2} + C\end{aligned}$$

$$\begin{aligned}\text{If } y(2) &= 3, \\ 3^3 &= \frac{3(2)^2}{2} + C \\ 27 &= 6 + C \\ 21 &= C \\ \text{So } y^3 &= \frac{3x^2}{2} + 21\end{aligned}$$

$$y = \sqrt[3]{\frac{3x^2}{2} + 21}$$

25. Find the particular solution of the equation.

$$\frac{dy}{dx} - 5y = 0, y(1) = 7$$

$$\frac{dy}{dx} = 5y \Rightarrow y = Ce^{5x}$$

When $y(1) = 7,$

$$7 = Ce^{5 \cdot 1}$$

$$7e^{-5} = C$$

$$y = 7e^{-5} e^{5x}$$

$$= 7e^{5x-5}$$

$$y = \underline{7e^{5x-5}}$$

26. The rate of change of the population $N(t)$ of a sample of bacteria is directly proportional to the number of bacteria, so $N'(t) = kN$, where time t is measured in minutes. Initially, there are 270 bacteria present. If the number of bacteria after 7 minutes is 770, find the growth rate k in terms of minutes. Round to four decimal places.

$$N' = kN \Rightarrow N(t) = Ce^{k \cdot t}$$

$$N(0) = 270 \Rightarrow 270 = Ce^{k \cdot 0}$$

$$270 = C \Rightarrow N(t) = 270e^{k \cdot t}$$

$$N(7) = 770 \Rightarrow 770 = 270e^{k \cdot 7}$$

$$\frac{770}{270} = e^{7k}$$

$$\ln\left(\frac{77}{27}\right) = 7k \quad k = \underline{0.1497}$$

$$k = \frac{1}{7} \ln\left(\frac{77}{27}\right)$$

27. What is the integrating factor of the following differential equation?

$$y' + \underbrace{\left(\frac{2x+3}{x}\right)}_{P(x)} y = 10 \ln(x)$$

$$\begin{aligned} u(x) &= \exp\left[\int P(x) dx\right] \\ &= \exp\left[\int \frac{2x+3}{x} dx\right] \\ &= \exp\left[\int 2 + \frac{3}{x} dx\right] \\ &= \exp\left[2x + 3 \ln(x)\right] \\ &= e^{2x} e^{3 \ln(x)} \end{aligned} \quad \left. \begin{aligned} &= e^{2x} e^{\ln(x^3)} \\ &= x^3 e^{2x} \end{aligned} \right\} u(x) = \underline{x^3 e^{2x}}$$

28. Find the general solution of the following differential equation.

$$P(x) = \frac{4}{x} \quad Q(x) = 2x + 20$$

$$\frac{dy}{dx} + \frac{4y}{x} = 2x + 20$$

$$\begin{aligned} u(x) &= \exp\left[\int P(x) dx\right] \\ &= \exp\left[\int \frac{4}{x} dx\right] \\ &= \exp\left[4 \ln(x)\right] \\ &= e^{\ln(x^4)} \\ &= x^4 \end{aligned} \quad \begin{aligned} y \cdot u(x) &= \int Q(x) \cdot u(x) dx + C \\ y \cdot x^4 &= \int (2x+20) x^4 dx + C \\ y \cdot x^4 &= \int (2x^5 + 20x^4) dx + C \\ y \cdot x^4 &= \frac{2x^6}{6} + \frac{20x^5}{5} + C \\ y &= \frac{x^2}{3} + 4x + \frac{C}{x^4} \end{aligned}$$

$$y = \underline{\frac{x^2}{3} + 4x + \frac{C}{x^4}}$$

29. Compute the following sum.

$$\sum_{n=0}^{\infty} \left(\frac{(-1)^n}{3^n} + \frac{2^{n+1}}{3^n} \right)$$

$$= \sum_{n=0}^{\infty} \left(\frac{(-1)^n}{3^n} + 2 \cdot \frac{2^n}{3^n} \right)$$

$$= \sum_{n=0}^{\infty} \left(\left(\frac{-1}{3} \right)^n + 2 \left(\frac{2}{3} \right)^n \right)$$

$$= \frac{1}{1 - (-1/3)} + \frac{2}{1 - 2/3}$$

$$= \frac{1}{4/3} + \frac{2}{1/3}$$

$$= \frac{3}{4} + 6$$

Answer: _____

27/4

30. Evaluate the sum of the following infinite series.

$$\sum_{n=1}^{\infty} \frac{4(3)^{n-1}}{5^n}$$

$$= \frac{4(3)^{1-1}}{5^1} + \frac{4(3)^{2-1}}{5^2} + \frac{4(3)^{3-1}}{5^3} + \dots$$

$$= \frac{4}{5} + \frac{4}{5^2}(3) + \frac{4(3)^2}{5^3} + \dots$$

$$= \frac{4}{5} \left(1 + \frac{3}{5} + \left(\frac{3}{5} \right)^2 + \dots \right)$$

$$= \frac{4}{5} \sum_{n=0}^{\infty} \left(\frac{3}{5} \right)^n$$

$$= \frac{4}{5} \cdot \frac{1}{1 - 3/5}$$

Answer: _____

2

31. Find the sum of the following series:

$$\begin{aligned} & \sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}} \\ &= \sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n} \cdot 3^1} = \sum_{n=0}^{\infty} \frac{1}{3} \cdot \frac{(-2)^n}{(3^2)^n} = \sum_{n=0}^{\infty} \frac{1}{3} \left(\frac{-2}{9} \right)^n \\ &= \frac{1}{3} \cdot \frac{1}{1 - (-2/9)} = \frac{1}{3} \cdot \frac{1}{11/9} = \frac{1}{3} \cdot \frac{9}{11} = \frac{3}{11} \end{aligned}$$

Answer: 3/11

32. Express $f(x) = \frac{x}{3+x}$ as a power series.

$$\begin{aligned} \frac{x}{1} \cdot \frac{1}{3+x} &= \frac{x}{3} \cdot \frac{1}{1+x/3} = \frac{x}{3} \cdot \frac{1}{1 - (-x/3)} \\ &= \frac{x}{3} \sum_{n=0}^{\infty} \left(\frac{-x}{3} \right)^n = \frac{x}{3} \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{3^n} \end{aligned}$$

$$\frac{x}{3+x} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{3^{n+1}}$$

33. Express $f(x) = \frac{x}{4+3x^2}$ as a power series.

$$\begin{aligned} \frac{x}{1} \cdot \frac{1}{4+3x^2} &= \frac{x}{4} \cdot \frac{1}{1+\frac{3}{4}x^2} = \frac{x}{4} \cdot \frac{1}{1-(-\frac{3}{4}x^2)} \\ &= \frac{x}{4} \sum_{n=0}^{\infty} \left(-\frac{3}{4}x^2\right)^n = \frac{x}{4} \sum_{n=0}^{\infty} \frac{(-1)^n 3^n (x^2)^n}{4^n} \\ &= \frac{x}{4} \sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^{2n}}{4^n} \end{aligned}$$

$$\frac{x}{4+3x^2} = \sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^{2n+1}}{4^{n+1}}$$

34. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$\begin{aligned} e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} \\ e^{-3x} &= \sum_{n=0}^{\infty} \frac{(-3x)^n}{n!} \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^n}{n!} \end{aligned}$$

$$\int e^{-3x} dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{n!} x^n dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{n!} \int x^n dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{n!} \cdot \frac{x^{n+1}}{n+1} + C$$

$$= \frac{(-1)^0 3^0}{0!} \cdot \frac{x^1}{1} + \frac{(-1)^1 3^1}{1!} \cdot \frac{x^2}{2} + \frac{(-1)^2 3^2}{2!} \cdot \frac{x^3}{3} + \dots$$

$$\int e^{-3x} dx = \boxed{\frac{x}{1} - \frac{3}{2}x^2 + \frac{3}{2}x^3 - \dots}$$

35. Find the Maclaurin representation of the following:

$$\begin{aligned} \cos(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} & \int \cos(\sqrt{x}) dx &= \int \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^n dx \\ \cos(\sqrt{x}) &= \sum_{n=0}^{\infty} \frac{(-1)^n (x^{1/2})^{2n}}{(2n)!} & &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \int x^n dx \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^n & &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot \frac{x^{n+1}}{n+1} \end{aligned}$$

$$\int \cos(\sqrt{x}) dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot \frac{x^{n+1}}{(n+1)}$$

36. Given $f(x, y) = 3x^3y^2 - x^2y^{1/3}$, evaluate $f(3, -8)$.

$$\begin{aligned} f(3, -8) &= 3(3)^3(-8)^2 - (3)^2(-8)^{1/3} \\ &= 5184 + 18 \end{aligned}$$

$$f(3, -8) = \underline{5202}$$

37. What do the level curves for the following function look like?

$$f(x, y) = 12 \ln(6(x-3)^2 + 6(y-2)^2) = C$$

- (a) Lines
- (b) Parabolas
- (c) Circles
- (d) Point at the origin
- (e) Ellipses
- (f) Hyperbolas

$$\ln(6(x-3)^2 + 6(y-2)^2) = \frac{C}{12} = C$$

$$6(x-3)^2 + 6(y-2)^2 = e^C = C$$

$$(x-3)^2 + (y-2)^2 = \frac{C}{6} = C$$

38. What do the level curves for the following function look like?

$$f(x, y) = \sqrt{y + 4x^2} = C$$

- (a) Lines
- (b) Parabolas
- (c) Circles
- (d) Point at the origin
- (e) Ellipses
- (f) Hyperbolas

$$y + 4x^2 = C^2 = C$$

$$y = -4x^2 + C$$

39. What do the level curves for the following function look like?

$$f(x, y) = \ln(y - e^{5x}) = C$$

- (a) Increasing exponential functions
- (b) Rational Functions with x-axis symmetry
- (c) Natural logarithm functions
- (d) Decreasing exponential functions
- (e) Rational Functions with y-axis symmetry

$$y = e^{5x} + C$$

40. What do the level curves for the following function look like?

$$f(x, y) = \cos(y + 4x^2) = C$$

- (a) Lines
- (b) Parabolas
- (c) Circles
- (d) Point at the origin
- (e) Ellipses
- (f) Hyperbolas

$$y + 4x^2 = \cos^{-1}(C) = C$$

$$y = -4x^2 + C$$

41. Given $f(x, y) = x \sin(xy^2)$, evaluate $f_y(3, 7)$. Round to 4 decimal places.

$$\begin{aligned} f_y(x, y) &= \frac{\partial}{\partial y} (x \sin(xy^2)) \\ &= x \frac{\partial}{\partial y} (\sin(xy^2)) \\ &= x \cos(xy^2) \frac{\partial}{\partial y} (xy^2) \\ &= x \cos(xy^2) \cdot x \frac{\partial}{\partial y} (y^2) \\ &= x^2 \cos(xy^2) \cdot (2y) \\ &= 2x^2 y \cos(xy^2) \end{aligned}$$

$$f_y(3, 7) = \underline{-99.9352}$$

42. Find the first order partial derivatives of $f(x, y) = xe^{xy}$

$$\begin{aligned} f_y &= \frac{\partial}{\partial y} (xe^{xy}) \\ &= x \frac{\partial}{\partial y} (e^{xy}) \\ &= x e^{xy} \frac{\partial}{\partial y} (xy) \\ &= x e^{xy} \cdot x \end{aligned}$$

$$\begin{aligned} f_x &= \frac{\partial}{\partial x} (xe^{xy}) \\ &= \frac{\partial}{\partial x} (x) e^{xy} + x \frac{\partial}{\partial x} (e^{xy}) \\ &= e^{xy} + x e^{xy} \frac{\partial}{\partial x} (xy) \\ &= e^{xy} + x y e^{xy} \\ &= (1 + xy) e^{xy} \end{aligned}$$

$$f_x(x, y) = \underline{(1 + xy) e^{xy}}$$

$$f_y(x, y) = \underline{x^2 e^{xy}}$$

43. For the given function $f(x, y)$, find $f_x(x, y)$.

$$f(x, y) = 5 \cos(x^7 y^8)$$

$$\begin{aligned} f_x &= \frac{\partial}{\partial x} (5 \cos(x^7 y^8)) \\ &= -5 \sin(x^7 y^8) \frac{\partial}{\partial x} (x^7 y^8) \\ &= -5 \sin(x^7 y^8) \cdot y^8 \frac{\partial}{\partial x} (x^7) \\ &= -5 y^8 \sin(x^7 y^8) \cdot 7x^6 \end{aligned}$$

$$f_x(x, y) = \underline{-35 x^6 y^8 \sin(x^7 y^8)}$$

44. Given the function $f(x, y) = x^3 y^2 - 3x + 5y - 5x^2 y^3$, compute $f_{xx}(x, y)$

$$\begin{aligned} f_x &= \frac{\partial}{\partial x} (x^3 y^2 - 3x + 5y - 5x^2 y^3) \\ &= 3x^2 y^2 - 3 + 0 - 5 \cdot 2x y^3 \\ &= 3x^2 y^2 - 3 - 10xy^3 \end{aligned}$$

$$\begin{aligned} f_{xx} &= \frac{\partial}{\partial x} (3x^2 y^2 - 3 - 10xy^3) \\ &= 3 \cdot 2x y^2 + 0 - 10y^3 \end{aligned}$$

$$f_{xx}(x, y) = \underline{6xy^2 - 10y^3}$$

45. Given the function $f(x, y) = x^3 \sin(y)$, compute $f_{xy}(2, 0)$

$$\begin{aligned} f_x &= \frac{\partial}{\partial x} (x^3 \sin(y)) \\ &= \sin(y) \frac{\partial}{\partial x} (x^3) \\ &= \sin(y) \cdot 3x^2 \end{aligned}$$

$$\begin{aligned} f_{xy}(2, 0) &= 3(2)^2 \cos(0) \\ &= 12 \end{aligned}$$

$$\begin{aligned} f_{xy} &= \frac{\partial}{\partial y} (\sin(y) \cdot 3x^2) \\ &= 3x^2 \frac{\partial}{\partial y} (\sin(y)) \\ &= 3x^2 \cos(y) \end{aligned}$$

$$f_{xy}(2, 0) = \underline{12}$$

46. For the function $f(x, y)$, find $f_{xy}(\pi, 2)$.

$$f(x, y) = 8y^5 \sin(x)$$

$$\begin{aligned} f_x &= \frac{\partial}{\partial x} (8y^5 \sin(x)) \\ &= 8y^5 \frac{\partial}{\partial x} (\sin(x)) \\ &= 8y^5 \cos(x) \end{aligned}$$

$$\begin{aligned} f_{xy}(\pi, 2) &= 40 \cos(\pi) \cdot 2^4 \\ &= -640 \end{aligned}$$

$$\begin{aligned} f_{xy} &= \frac{\partial}{\partial y} (8y^5 \cos(x)) \\ &= \cos(x) \frac{\partial}{\partial y} (8y^5) \\ &= \cos(x) \cdot 40y^4 \end{aligned}$$

$$f_{xy}(\pi, 2) = \underline{-640}$$

47. At what point (x, y) does the function $f(x, y)$ have a local minimum?

$$f(x, y) = 7x^2 - xy + 5y^2 + 75x + 84y + 2$$

$$f_x = 14x - y + 0 + 75 + 0 + 0$$

$$= 14x - y + 75$$

$$f_y = 0 - x + 10y + 0 + 84$$

$$= -x + 10y + 84$$

$$\begin{cases} 14x - y + 75 = 0 & \textcircled{1} \\ -x + 10y + 84 = 0 & \textcircled{2} \end{cases}$$

Multiply $\textcircled{1}$ by 10 and add w/ $\textcircled{2}$

$$\begin{array}{r} 140x - 10y + 750 = 0 \\ + \quad -x + 10y + 84 = 0 \\ \hline 139x + 834 = 0 \\ x = 6 \end{array}$$

$$\begin{array}{l|l} \text{Plug } x=6 \text{ into } \textcircled{1}. & 159 - y = 0 \\ 14(6) - y + 75 = 0 & y = 159 \\ 84 - y + 75 = 0 & \end{array}$$

$$f_{xx} = 14 \quad f_{xy} = -1$$

Local min occurs at $(6, 159)$

$$f_{yy} = 10 \quad \Rightarrow D > 0 + f_{xx} < 0 \text{ for all pts } \uparrow$$

48. Given the information below, which critical point(s) (a, b) would be classified as a relative maximum?

| (a, b) | $f_{xx}(a, b)$ | $f_{yy}(a, b)$ | $f_{xy}(a, b)$ |
|------------|----------------|----------------|----------------|
| $(7, 8)$ | -5 | -5 | 10 |
| $(-8, -1)$ | -4 | -7 | -2 |
| $(1, 7)$ | -10 | -1 | 6 |

$$D = f_{xx} f_{yy} - (f_{xy})^2$$

$$D(7, 8) = (-5)(-5) - 10^2 < 0 \Rightarrow \text{saddle pt}$$

$$\left. \begin{array}{l} D(-8, -1) = (-4)(-7) - (-2)^2 > 0 \\ f_{xx}(-8, -1) < 0 \end{array} \right\} \Rightarrow \text{rel max}$$

$$D(1, 7) = (-10)(-1) - (6)^2 < 0 \Rightarrow \text{saddle pt}$$

Answer: $(-8, -1)$

$$D = f_{xx}f_{yy} - (f_{xy})^2$$

49. Given the table below,

| (a, b) | $f_{xx}(a, b)$ | $f_{yy}(a, b)$ | $f_{xy}(a, b)$ |
|-----------|----------------|----------------|----------------|
| $(9, 4)$ | -1 | -1 | -1 |
| $(-2, 2)$ | 4 | 3 | -4 |
| $(4, 5)$ | 8 | 5 | 6 |

Which statements are true?

- I. $f(x, y)$ has exactly 1 saddle points ✓
- II. $f(x, y)$ has exactly 1 relative minimum ✓
- III. $f(x, y)$ has exactly 1 relative maximum
- IV. $f(x, y)$ has exactly 1 inconclusive critical point ✓

$$D(9, 4) = (-1)(-1) - (-1)^2 = 0 \rightarrow \text{inconclusive}$$

$$D(-2, 2) = (4)(3) - (-4)^2 < 0 \rightarrow \text{saddle}$$

$$D(4, 5) = (8)(5) - 6^2 > 0 \left. \begin{array}{l} \text{rel min} \\ f_{xx}(4, 5) = 8 > 0 \end{array} \right\}$$

Answer: I, II, IV

50. The critical points for a function $f(x, y)$ are $(0, 0)$ and $(8, 4)$. Given that the partial derivatives of $f(x, y)$ are

$$f_x(x, y) = 3x - 6y \quad f_y(x, y) = 3y^2 - 6x$$

Classify each critical point as a maximum, minimum, or saddle point.

Critical Pts: $(0, 0), (8, 4)$

$$f_x = 3x - 6y$$

$$f_y = 3y^2 - 6x$$

$$f_{xx} = 3$$

$$f_{yy} = 6y$$

$$f_{xy} = -6$$

$$D(0, 0) = -36 < 0 \text{ saddle}$$

$$D(8, 4) = 18(4) - 36 > 0 \left. \begin{array}{l} \text{rel} \\ \text{min} \end{array} \right\} \\ f_{xx} = 3 > 0$$

$$D = f_{xx}f_{yy} - (f_{xy})^2$$

$$= 3(6y) - (-6)^2$$

$$= 18y - 36$$

$(0, 0)$ is saddle pt

$(8, 4)$ is relative min

51. Classify the critical points of the function $f(x, y)$ given the partial derivatives

$$f_x(x, y) = x - 2y \quad f_y(x, y) = y^2 - 2x$$

- A) 2 Local Minima
- B) 2 Saddle Points
- C) 1 Saddle Point and 1 Local Minimum
- D) 1 Saddle Point and 1 Local Maximum
- E) 1 Local Maximum and 1 Local Minimum
- F) 2 Local Maxima

$$\begin{cases} x - 2y = 0 & \textcircled{1} \\ y^2 - 2x = 0 & \textcircled{2} \end{cases}$$

Solve ① for x .
 $x = 2y$

Plug $x = 2y$ into ②

$$\begin{aligned} y^2 - 2(2y) &= 0 \\ y^2 - 4y &= 0 \\ y(y - 4) &= 0 \\ y &= 0, 4 \end{aligned}$$

Plug $y = 0, 4$ into $x = 2y$

$$\begin{aligned} \underline{y=0}: x &= 0 \rightarrow (0, 0) \\ \underline{y=4}: x &= 8 \rightarrow (8, 4) \end{aligned}$$

$$f_x = x - 2y$$

$$f_{xx} = 1$$

$$f_{xy} = -2$$

$$f_y = y^2 - 2x$$

$$f_{yy} = 2y$$

$$D = f_{xx}f_{yy} - (f_{xy})^2$$

$$= 1(2y) - (-2)^2$$

$$= 2y - 4$$

$$D(0, 0) = -4 < \text{saddle}$$

$$D(8, 4) > 0 + f_{xx} = 1 > 0 \Rightarrow \text{rel min}$$

52. Find the minimum of the function using LaGrange Multipliers of the function $f(x, y) = x^2 + 2y^2$ subject to the constraint $x^2 + y^2 = 1$.

$$f(x, y) = x^2 + 2y^2$$

$$f_x = 2x$$

$$f_y = 4y$$

$$g(x, y) = x^2 + y^2$$

$$g_x = 2x$$

$$g_y = 2y$$

Plug $\lambda = 1$ into ②

$$4y = 2y$$

$$2y = 0$$

$$y = 0$$

$$\begin{cases} 2x = \lambda 2x & \textcircled{1} \\ 4y = \lambda 2y & \textcircled{2} \\ x^2 + y^2 = 1 & \textcircled{3} \end{cases}$$

Solve ①,

$$2x = \lambda 2x$$

$$2x - \lambda 2x = 0$$

$$2x(1 - \lambda) = 0$$

$$x = 0, \lambda = 1$$

Plug $y = 0$ into ③

$$x^2 = 1$$

$$x = \pm 1$$

Critical Pts:

$$(1, 0), (-1, 0)$$

$$f(x, y) = x^2 + 2y^2$$

$$f(\pm 1, 0) = 1$$

$$f(0, \pm 1) = 2$$

Minimum Value = 1

Plug $x = 0$ into ③

$$\begin{cases} y^2 = 1 \\ y = \pm 1 \end{cases} \left. \begin{array}{l} \text{Critical Pts: } (0, 1) \\ (0, -1) \end{array} \right\}$$

53. Find the maximum value of the function $f(x, y) = 18x - 19y^2$ subject to the constraint $x^2 + 19y^2 = 81$.

$$f(x, y) = 18x - 19y^2 \quad g(x, y) = x^2 + 19y^2 \quad \left\{ \begin{array}{l} \text{Plug } y=0 \text{ into } \textcircled{3} \\ x^2 = 81 \\ x = \pm 9 \\ \text{Critical Pt: } (9, 0), (-9, 0) \end{array} \right.$$

$$f_x = 18 \quad g_x = 2x$$

$$f_y = -38y \quad g_y = 38y$$

$$\left\{ \begin{array}{l} 18 = \lambda 2x \quad \textcircled{1} \\ -38y = \lambda 38y \quad \textcircled{2} \\ x^2 + 19y^2 = 81 \quad \textcircled{3} \end{array} \right.$$

Solve $\textcircled{2}$.

$$38y - \lambda 38y = 0$$

$$38y(1 + \lambda) = 0$$

$$y = 0, \lambda = -1$$

Plug $\lambda = -1$ into $\textcircled{1}$

$$18 = -2x$$

$$x = -9$$

Plug $x = -9$ into $\textcircled{3}$

$$(-9)^2 + 19y^2 = 81$$

$$18y^2 = 0$$

$$y = 0$$

Critical Pt
 $(-9, 0)$

$$f(x, y) = 18x - 19y^2$$

$$f(9, 0) = 162$$

$$f(-9, 0) = -162$$

Maximum Value = 162

54. A factory can produce a chocolate bar with a weight of $W(x, y) = \frac{xy}{100}$ with the weight W in ounces and x and y are the percentages of cocoa and sugar respectively. The percentage of cocoa and sugar are constrained to $2x + y = 75$. What is the weight, in ounces, of the largest chocolate bar that can be produced? Round to 2 decimal places.

$$W(x, y) = \frac{xy}{100} \quad g(x, y) = 2x + y \quad \left\{ \begin{array}{l} \text{Plug } y = 2x \text{ into } 2x + y = 75 \\ 2x + 2x = 75 \\ 4x = 75 \\ x = 75/4 \end{array} \right.$$

$$W_x = y/100 \quad g_x = 2$$

$$W_y = x/100 \quad g_y = 1$$

Hence $y = 75/2$

$$W\left(\frac{75}{4}, \frac{75}{2}\right) \approx 7.03$$

$$\left\{ \begin{array}{l} \frac{y}{100} = 2\lambda \quad \textcircled{1} \\ \frac{x}{100} = \lambda \quad \textcircled{2} \\ 2x + y = 75 \quad \textcircled{3} \end{array} \right.$$

Plug $\textcircled{2}$ into $\textcircled{1}$

$$\frac{y}{100} = 2\left(\frac{x}{100}\right)$$

$$y = 2x$$

Weight of Largest Chocolate Bar = 7.03

55. Evaluate the double integral

$$\int_0^{\pi/2} \int_0^1 16y^3 \cos(x) dy dx$$

$$\begin{aligned} & \int_{x=0}^{\pi/2} \left(\int_{y=0}^1 16y^3 \cos(x) dy \right) dx \\ &= \int_{x=0}^{\pi/2} \cos(x) \left(\int_{y=0}^1 16y^3 dy \right) dx \\ &= \int_{x=0}^{\pi/2} \cos(x) \left(\frac{16y^4}{4} \right) \Big|_{y=0}^{y=1} dx \\ &= \int_{x=0}^{\pi/2} \cos(x) \cdot 4 dx \\ &= 4 \sin(x) \Big|_{x=0}^{x=\pi/2} \\ &= 4 \sin\left(\frac{\pi}{2}\right) - 4 \sin(0) \end{aligned}$$

$$\int_0^1 \int_0^{\pi/2} 16y^3 \cos(x) dy dx = \underline{4}$$

56. Evaluate the double integral

$$\int_0^7 \int_0^{\pi/2} 20y^4 \cos(x) dx dy$$

$$\begin{aligned} & \int_{y=0}^7 \left(\int_{x=0}^{\pi/2} 20y^4 \cos(x) dx \right) dy \\ &= \int_{y=0}^7 20y^4 \left(\int_{x=0}^{\pi/2} \cos(x) dx \right) dy \\ &= \int_{y=0}^7 20y^4 \left(\sin(x) \right) \Big|_{x=0}^{x=\pi/2} dy \\ &= \int_{y=0}^7 20y^4 \cdot 1 dy \\ &= \frac{20y^5}{5} \Big|_{y=0}^{y=7} \\ &= 4(7)^5 \end{aligned}$$

$$\int_0^7 \int_0^{\pi/2} 20y^4 \cos(x) dx dy = \underline{67228}$$

57. Evaluate the definite integral.

$$\int_0^4 \int_3^x \frac{6x}{y^2} dy dx$$

$$\begin{aligned} & \int_{x=0}^{x=4} \left(\int_{y=3}^{y=x} 6xy^{-2} dy \right) dx & \left. \begin{aligned} & = \left(-6x + \frac{2x^2}{2} \right) \Big|_{x=0}^{x=4} \\ & = (x^2 - 6x) \Big|_{x=0}^{x=4} \\ & = -8 \end{aligned} \right\} \\ & = \int_{x=0}^{x=4} 6x \left(\int_{y=3}^{y=x} y^{-2} dy \right) dx \\ & = \int_{x=0}^{x=4} 6x \left(\frac{y^{-1}}{-1} \Big|_{y=3}^{y=x} \right) dx \\ & = \int_{x=0}^{x=4} 6x \left(-\frac{1}{y} \Big|_{y=3}^{y=x} \right) dx \\ & = \int_{x=0}^{x=4} 6x \left(-\frac{1}{x} + \frac{1}{3} \right) dx \\ & = \int_{x=0}^{x=4} (-6 + 2x) dx \end{aligned}$$

$$\int_0^4 \int_3^x \frac{6x}{y^2} dy dx = \underline{\underline{-8}}$$

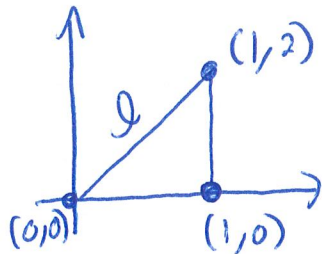
58. Evaluate the definite integral.

$$\int_0^6 \int_2^x 30x dy dx$$

$$\begin{aligned} & \int_{x=0}^{x=6} \left(\int_{y=2}^{y=x} 30x dy \right) dx & \left. \begin{aligned} & = (10x^3 - 30x^2) \Big|_{x=0}^{x=6} \\ & = 1080 \end{aligned} \right\} \\ & = \int_{x=0}^{x=6} 30x \left(\int_{y=2}^{y=x} dy \right) dx \\ & = \int_{x=0}^{x=6} 30x \left(y \Big|_{y=2}^{y=x} \right) dx \\ & = \int_{x=0}^{x=6} 30x (x - 2) dx \\ & = \int_{x=0}^{x=6} (30x^2 - 60x) dx \\ & = \left(\frac{30x^3}{3} - \frac{60x^2}{2} \right) \Big|_{x=0}^{x=6} \end{aligned}$$

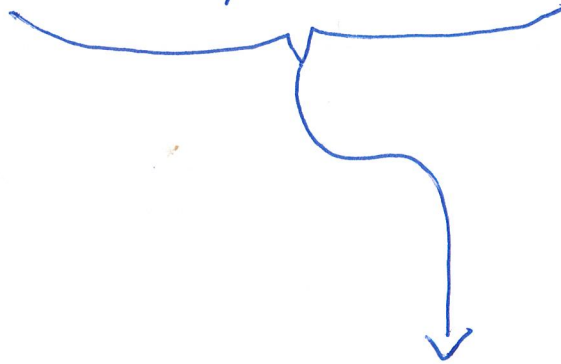
$$\int_0^6 \int_2^x 30x dy dx = \underline{\underline{1080}}$$

59. Find the bounds for the integral $\iint_R f(x, y) dA$ where R is a triangle with vertices $(0,0)$, $(1,0)$, and $(1,2)$.



Equation of d : $m = \frac{2-0}{1-0} = 2 \Rightarrow y = 2x$

$$\int_{x=0}^1 \int_{y=0}^{y=2x} f(x, y) dy dx$$



Answer: _____

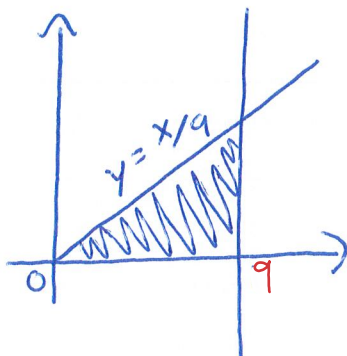
60. Switch the order of integration for the following integral

$dy dx \Rightarrow$ Top + Bottom

$$\int_0^1 \int_{9y}^9 f(x, y) dx dy$$

$$\int_{y=0}^1 \int_{x=9y}^{x=9} f(x, y) dx dy$$

Region is $y=0, y=1, x=9y, x=9$
 \downarrow
 $y = \frac{x}{9}$



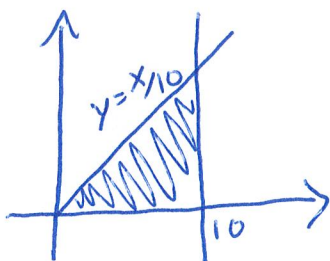
Answer: $\int_{x=0}^9 \int_{y=0}^{y=x/9} f(x, y) dy dx$

61. Switch the order of integration on the follow integral

$$\int_0^1 \int_{10y}^{10} f(x,y) dx dy$$

$$\int_{y=0}^{y=1} \int_{x=10y}^{x=10} f(x,y) dx dy$$

Region $y=0, y=1, x=10y, x=10$
 \uparrow
 $y = x/10$



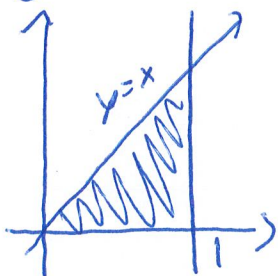
Answer: $\int_{x=0}^{x=10} \int_{y=0}^{y=x/10} f(x,y) dy dx$

62. Evaluate the double integral

$$\int_0^1 \int_y^1 2e^{x^2} dx dy = \int_{y=0}^{y=1} \int_{x=y}^{x=1} 2e^{x^2} dx dy$$

(Hint: Change the order of integration)

Region $y=0, y=1, x=y, x=1$



$$\int_{x=0}^{x=1} \left(\int_{y=0}^{y=x} 2e^{x^2} dy \right) dx$$

$$= \int_{x=0}^{x=1} 2e^{x^2} \left(\int_{y=0}^{y=x} dy \right) dx$$

$$= \int_{x=0}^{x=1} 2e^{x^2} (y) \Big|_{y=0}^{y=x} dx$$

$$= \int_{x=0}^{x=1} 2e^{x^2} \cdot x dx$$

$$\frac{u = x^2}{du = 2x dx} \int e^u du = e^u$$

$$= e^{x^2} \Big|_{x=0}^{x=1}$$

$$= e^1 - e^0$$

$$\int_0^1 \int_y^1 2e^{x^2} dx dy = \underline{e-1}$$

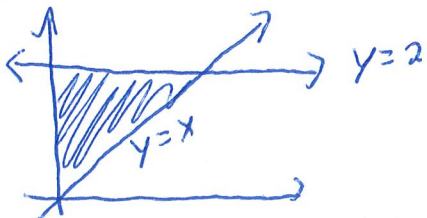
$dx dy \Rightarrow$ Right + Left

63. Evaluate the double integral

$$\int_0^2 \int_x^2 4e^{y^2} dy dx = \int_{x=0}^{x=2} \int_{y=x}^{y=2} 4e^{y^2} dy dx$$

(Hint: Change the order of integration)

Region $x=0, x=2, y=x, y=2$



$$\begin{aligned} & \int_{y=0}^{y=2} \left(\int_{x=0}^{x=y} 4e^{y^2} dx \right) dy \\ &= \int_{y=0}^{y=2} 4e^{y^2} \left(\int_{x=0}^{x=y} dx \right) dy \end{aligned}$$

$$= \int_{y=0}^{y=2} 4e^{y^2} \left(x \right) \Big|_{x=0}^{x=y} dy$$

$$= \int_{y=0}^{y=2} 4e^{y^2} \cdot y dy$$

$$\begin{aligned} & \frac{u=y^2}{du=2y dy} \int 4y e^u \frac{du}{2y} = \int 2e^u du \\ & \frac{du}{2y} = dy \quad \left[= 2e^u = 2e^{y^2} \right]_{y=0}^{y=2} \end{aligned}$$

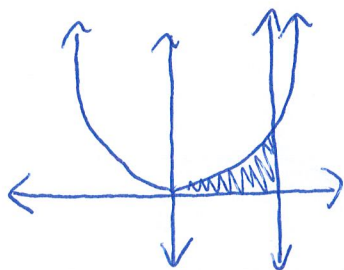
$$\int_0^2 \int_x^2 4e^{y^2} dy dx = \underline{2e^4 - 2}$$

64. Evaluate the double integral

$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3) dx dy = \int_{y=0}^{y=1} \int_{x=\sqrt{y}}^{x=1} \sin(x^3) dx dy$$

(Hint: Change the order of integration)

Region $y=0, y=1, x=\sqrt{y}, x=1$



$$\begin{aligned} & \int_{x=0}^{x=1} \left(\int_{y=0}^{y=x^2} \sin(x^3) dy \right) dx \\ &= \int_{x=0}^{x=1} \sin(x^3) \left(\int_{y=0}^{y=x^2} dy \right) dx \\ &= \int_{x=0}^{x=1} \sin(x^3) \left(y \right) \Big|_{y=0}^{y=x^2} dx \end{aligned}$$

$$= \int_{x=0}^{x=1} x^2 \sin(x^3) dx$$

$$\frac{u=x^3}{du=3x^2 dx} \int x^2 \sin(u) \frac{du}{3x^2}$$

$$\frac{du}{3x^2} = dx$$

$$= \frac{1}{3} \int \sin(u) du = -\frac{1}{3} \cos(u)$$

$$= -\frac{1}{3} \cos(x^3) \Big|_{x=0}^{x=1}$$

$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3) dx dy = \underline{-\frac{1}{3} \cos(1) + \frac{1}{3}}$$