

Please show **all** your work! Answers without supporting work will not be given credit.
Write answers in spaces provided.

Name: _____

1. Evaluate the indefinite integral

$$\int 12x(x^2 + 1)^{100} dx$$

$$\int 12x(x^2 + 1)^{100} dx = \underline{\hspace{10cm}}$$

2. Evaluate the indefinite integral

$$\int \frac{\ln(x^2)}{x} dx$$

$$\int \frac{\ln(x^2)}{x} dx = \underline{\hspace{10cm}}$$

5. Which of the following is a partial fraction decomposition of

$$f(x) = \frac{2x + 1}{(x - 8)(x + 8)(x^2 + 36)}$$

- (A) $\frac{A}{x^2 - 64} + \frac{B}{x^2 + 36}$
- (B) $\frac{A}{x - 8} + \frac{B}{x + 8} + \frac{C}{x^2 + 36}$
- (C) $\frac{A}{x - 8} + \frac{B}{x + 8} + \frac{C}{x - 6} + \frac{D}{x + 6}$
- (D) $\frac{Ax}{x^2 - 64} + \frac{Bx}{x^2 + 36}$
- (E) $\frac{A}{x - 8} + \frac{B}{x + 8} + \frac{Cx + D}{x^2 + 36}$
- (F) $\frac{A}{(x - 8)^2} + \frac{B}{(x + 8)^2} + \frac{C}{x + 6} + \frac{D}{(x + 6)^2}$

6. Which of the following statements are true?

- (i) $\int_1^\infty \sqrt{x} \, dx$ is an improper integral that diverges.
- (ii) $\int_1^\infty \frac{1}{\sqrt{x}} \, dx$ is an improper integral that can converges.
- (iii) $\int_1^\infty \frac{1}{x} \, dx$ is an improper integral that can converges.
- (iv) $\int_1^\infty \frac{1}{x^2} \, dx$ is an improper integral that can converges.

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7. Calculate the area between the region bounded by the curves $y = -x$ and $y = 20 - x^2$.

Area = _____

8. Find the volume of the solid that results by revolving the region enclosed by the curves $y = \frac{4}{x}$, $y = 0$, $x = 1$, and $x = 4$ about the x -axis.

Volume = _____

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9. SET-UP the integral that can be used to calculate the volume of the solid obtained by revolving the region bounded by the curves $y = x^2$ and $y = 3x$ about the y -axis using cylindrical shells.

Volume = _____

10. SET-UP the integral that would calculate the volume of the solid produced by revolving the region bounded by the curves $y = x^4$ and $x = y^4$ about the x -axis using the washer method.

Volume = _____

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11. Calculate the volume of the solid obtained by revolving the region bounded by the curves $y = 2x$, $y = 0$, and $x = 4$ about the y -axis.

Volume = _____

12. Find the general solution of the first-order linear differential equation

$$\frac{dy}{dx} + \frac{5y}{x} = x^2 + 7$$

$y =$ _____

13. Find the sum of the geometric series

$$\sum_{n=0}^{\infty} (-1)^n \frac{3^{2n}}{4^{n+1}}$$

if it exists.

$$\sum_{n=0}^{\infty} (-1)^n \frac{3^{2n}}{4^{n+1}} = \underline{\hspace{10em}}$$

14. Which of the following is a function whose power series representation is given by the series

$$\sum_{n=0}^{\infty} \frac{5x^{n+2}}{7^{n+1}}$$

(A) $f(x) = \frac{5x^2}{1-7x}$

(B) $f(x) = \frac{5x^2}{1+7x}$

(C) $f(x) = \frac{5x}{1-7x}$

(D) $f(x) = \frac{5x^2}{7+x}$

(E) $f(x) = \frac{5x}{1+7x}$

(F) $f(x) = \frac{5x^2}{7-x}$

15. Estimate the value of the definite integral

$$\int_0^{0.4} \ln(1 + x^2) dx$$

using the first three non-zero terms of the Maclaurin series expansion of $f(x) = \ln(1 + x)$. Round to 6 decimal places.

$$\int_0^{0.4} \ln(1 + x^2) dx \approx \underline{\hspace{10em}}$$

16. The function $f(x, y)$ has the first partial derivatives shown below. Classify all of the extrema (local minima/maxima and saddle point(s)) of $f(x, y)$.

$$f_x(x, y) = x - y \qquad f_y(x, y) = y^3 - x$$

- (A) There are two saddle points and one local minimum
- (B) There are two local maxima and one local minimum
- (C) There are three saddle points
- (D) There are two local minima and one saddle point
- (E) There is one saddle point, one local maximum, and one local minimum
- (F) There are two saddle points and one local maximum

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17. A bakery sells a particular cupcake where the customer satisfaction is given by $S(x, y) = 6x^{3/2}y$, where x is the grams of sugar and y is the grams of spice. If the sugar and spice used must satisfy $9x + y = 4$, what is the maximum customer satisfaction that can be achieved? Round your answer to 2 decimal places. (Note the function is only defined when $x \geq 0$ and $y \geq 0$)

Maximum customer satisfaction = _____

18. Evaluate the definite integral

$$\int_0^{\sqrt{\pi/6}} \int_y^{\sqrt{\pi/6}} \cos(x^2) dx dy$$

(HINT: Change the order of integration.)

$$\int_0^{\sqrt{\pi/6}} \int_y^{\sqrt{\pi/6}} \cos(x^2) dx dy = \underline{\hspace{2cm}}$$