SP26_MA16020_Practice_Final_Exam_Pt2

Tuesday, April 28, 2026 12:04 PM

SP26_MA
16020_Pr...

Please show **all** your work! Answers without supporting work will not be given credit.
Write answers in spaces provided.

Name:____

1. Evaluate the indefinite integral

$$\int 12x(x^2+1)^{100}\,dx$$

$u=x^2+1$

$du=2x\,dx$

$\frac{du}{2x}=dx$

$$\int \overset{6}{\cancel{12x}}\,u^{100}\cdot\frac{du}{\cancel{2x}} = \int 6u^{100}\,du = \frac{6u^{101}}{101} = \frac{6(x^2+1)^{101}}{101}+C$$

$\int 12x(x^2+1)^{100}\,dx =$ $\frac{6(x^2+1)^{101}}{101}+C$

2. Evaluate the indefinite integral

$$\int \frac{\ln(x^2)}{x}\,dx = \int \frac{2\ln(x)}{x}\,dx$$

$u=\ln(x)$

$du=\frac{1}{x}dx$

$x\,du=dx$

$$\int \frac{2u}{x}\cdot x\,du$$

$$=\int 2u\,du$$

$$=\frac{2u^2}{2} = (\ln(x))^2+C$$

$\int \frac{\ln(x^2)}{x}\,dx =$ $(\ln(x))^2+C$

3. Evaluate the indefinite integral

$$\int 4xe^{9x}\,dx$$

$\underline{u = 4x}$ $\quad$ $\underline{dv = e^{9x}dx}$ $\quad uv - \int v\,du = \frac{4xe^{9x}}{9} - \int \frac{4}{9} e^{9x} dx$

$du = 4dx$ $\quad$ $v = \frac{e^{9x}}{9}$

$= \frac{4xe^{9x}}{9} - \frac{4}{9} \cdot \frac{e^{9x}}{9} + C$

$\int 4xe^{9x}\,dx =$ $\frac{4xe^{9x}}{9} - \frac{4e^{9x}}{81} + C$

4. The velocity of a car in the time period $0 \leq t \leq 3$ (in hours) is given by the function $v(t) = 55te^{-t/4}$ miles per hour. What was the distance that the car traveled in the first two hours? Round your answer to 2 decimal places.

$\int_0^2 55te^{-t/4}dt$ $\quad$ $\underline{u = 55t}$ $\quad$ $\underline{dv = e^{-t/4}dt}$ $\quad uv - \int v\,du$

$du = 55dt$ $\quad$ $v = -4e^{-t/4}$

$= 55t(-4)e^{-t/4} - \int 55(-4)e^{-t/4}dt$

$= -220te^{-t/4} + 220\int e^{-t/4}dt$

$= -220te^{-t/4} + 220e^{-t/4}(-4)\Big]_0^2$

$= -220te^{-t/4} - 880e^{-t/4}\Big]_0^2$

Answer: 79.38

5. Which of the following is a partial fraction decomposition of

$$f(x) = \frac{2x+1}{(x-8)(x+8)(x^2+36)}$$

(A) $\frac{A}{x^2-64} + \frac{B}{x^2+36}$

(B) $\frac{A}{x-8} + \frac{B}{x+8} + \frac{C}{x^2+36}$

(C) $\frac{A}{x-8} + \frac{B}{x+8} + \frac{C}{x-6} + \frac{D}{x+6}$

(D) $\frac{Ax}{x^2-64} + \frac{Bx}{x^2+36}$

(E) $\frac{A}{x-8} + \frac{B}{x+8} + \frac{Cx+D}{x^2+36}$

(F) $\frac{A}{(x-8)^2} + \frac{B}{(x+8)^2} + \frac{C}{x+6} + \frac{D}{(x+6)^2}$

6. Which of the following statements are true?

(i) $\int_1^\infty \sqrt{x}\,dx$ is an improper integral that diverges. ✓

(ii) $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$ is an improper integral that can converges. ✗

(iii) $\int_1^\infty \frac{1}{x}\,dx$ is an improper integral that can converges. ✗

(iv) $\int_1^\infty \frac{1}{x^2}\,dx$ is an improper integral that can converges. ✓

(i) $\int_1^\infty \sqrt{x}\,dx = \int_1^\infty x^{1/2}\,dx = \frac{2}{3}x^{3/2}\Big]_1^\infty = \infty \Rightarrow$ diverges

(ii) $\int_1^\infty \frac{1}{\sqrt{x}}\,dx = \int_1^\infty x^{-1/2}\,dx = 2x^{1/2}\Big]_1^\infty = \infty \rightarrow$ diverges

(iii) $\int_1^\infty \frac{1}{x}\,dx = \ln(x)\Big]_1^\infty = \infty \rightarrow$ diverges

(iv) $\int_1^\infty \frac{1}{x^2}\,dx = \frac{-1}{x}\Big]_1^\infty = 0 + 1 = 1 \rightarrow$ converge

7. Calculate the area between the region bounded by the curves $y = -x$ and $y = 20 - x^2$.

Bounds: $-x = 20 - x^2$

$x^2 - x - 20 = 0$

$(x-5)(x+4) = 0$

$x = 5, -4$

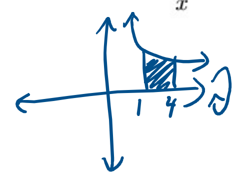
$$\int_{-4}^{5} 20 - x^2 - (-x)\, dx$$

$$= \int_{-4}^{5} (20 - x^2 + x)\, dx$$

$$= \left(20x - \frac{x^3}{3} + \frac{x^2}{2}\right)\Big]_{-4}^{5}$$

Area = 121.5

8. Find the volume of the solid that results by revolving the region enclosed by the curves $y = \frac{4}{x}$, $y = 0$, $x = 1$, and $x = 4$ about the x-axis.



$$V = \pi \int_{1}^{4} \left(\frac{4}{x}\right)^2 dx$$

$$= \pi \int_{1}^{4} \frac{16}{x^2}\, dx$$

$$= \pi \int_{1}^{4} 16x^{-2}\, dx$$

$$= \pi (-16)\, x^{-1} \Big]_{1}^{4}$$

$$= -\frac{16\pi}{x}\Big]_{1}^{4} = -\frac{16\pi}{4} - \left(-\frac{16\pi}{1}\right) = -4\pi + 16\pi = 12\pi$$

Volume = 12π

9. SET-UP the integral that can be used to calculate the volume of the solid obtained by revolving the region bounded by the curves $y = x^2$ and $y = 3x$ about the y-axis using cylindrical shells.

dx

Bounds: $x^2 = 3x$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0, 3$

$$V = 2\pi \int_0^3 x(3x - x^2)\,dx$$

$$= 2\pi \int_0^3 3x^2 - x^3\,dx$$

Volume = $2\pi \int_0^3 (3x^2 - x^3)\,dx$

10. SET-UP the integral that would calculate the volume of the solid produced by revolving the region bounded by the curves $y = x^4$ and $x = y^4$ about the x-axis using the washer method.

$y = x^{1/4}$

dx

Bounds: $x^4 = x^{1/4}$

$(x^4)^4 = (x^{1/4})^4$

$x^{16} = x$

$x^{16} - x = 0$

$x(x^{15} - 1) = 0$

$x = 0, 1$

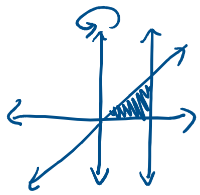
$$V = \pi \int_0^1 (x^{1/4})^2 - (x^4)^2\,dx$$

$$= \pi \int_0^1 x^{1/2} - x^8\,dx$$

Volume = $\pi \int_0^1 x^{1/2} - x^8\,dx$

11. Calculate the volume of the solid obtained by revolving the region bounded by the curves $y = 2x$, $y = 0$, and $x = 4$ about the y-axis.

Shells is easier. $\Rightarrow dx$



$$V = 2\pi \int_0^4 x(2x)\,dx$$
$$= 4\pi \int_0^4 x^2\,dx$$
$$= 4\pi \left.\frac{x^3}{3}\right]_0^4 = \frac{256\pi}{3}$$

Volume = $256\pi/3$

12. Find the general solution of the first-order linear differential equation

$$\frac{dy}{dx} + \frac{5y}{x} = x^2 + 7$$

$$u(x) = \exp\left[\int \frac{5}{x}\,dx\right] = \exp\left[5\ln(x)\right] = \exp\left[\ln(x^5)\right] = x^5$$

$$y\,u(x) = \int u(x)\,Q(x)\,dx$$
$$y\,x^5 = \int x^5(x^2+7)\,dx$$
$$y\,x^5 = \int x^7 + 7x^5\,dx$$
$$y\,x^5 = \frac{x^8}{8} + \frac{7x^6}{6} + C$$
$$y = \frac{x^3}{8} + \frac{7}{6}x + \frac{C}{x^5}$$

y = $\frac{x^3}{8} + \frac{7x}{6} + \frac{C}{x^5}$

13. Find the sum of the geometric series

$$\sum_{n=0}^{\infty}(-1)^n\frac{3^{2n}}{4^{n+1}}$$

$= \sum_{n=0}^{\infty} \frac{(-1)^n(3^2)^n}{4\cdot 4^n} = \frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{-3^2}{4}\right)^n$

$= \frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{-9}{4}\right)^n$

but

$|-9/4| \not< 1$

if it exists.

$\sum_{n=0}^{\infty}(-1)^n\frac{3^{2n}}{4^{n+1}} =$ diverges

14. Which of the following is a function whose power series representation is given by the series

$$\sum_{n=0}^{\infty}\frac{5x^{n+2}}{7^{n+1}}$$

$= \sum_{n=0}^{\infty}\frac{5x^2\,x^n}{7\cdot 7^n} = \sum_{n=0}^{\infty}\frac{5x^2}{7}\left(\frac{x}{7}\right)^n$

$= \frac{5x^2}{7}\cdot\frac{1}{1-x/7}$

$= \frac{5x^2}{7-x}$

.

(A) $f(x) = \frac{5x^2}{1-7x}$

(B) $f(x) = \frac{5x^2}{1+7x}$

(C) $f(x) = \frac{5x}{1-7x}$

(D) $f(x) = \frac{5x^2}{7+x}$

(E) $f(x) = \frac{5x}{1+7x}$

(F) $f(x) = \frac{5x^2}{7-x}$

15. Estimate the value of the definite integral

$$\int_0^{0.4} \ln(1+x^2)\,dx$$

using the first three non-zero terms of the Maclaurin series expansion of $f(x) = \ln(1+x)$. Round to 6 decimal places.

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3}$

$\ln(1+x^2) = x^2 - \frac{(x^2)^2}{2} + \frac{(x^2)^3}{3} = x^2 - \frac{x^4}{2} + \frac{x^6}{3}$

$\int_0^{0.4} x^2 - \frac{x^4}{2} + \frac{x^6}{3}\,dx = \left.\frac{x^3}{3} - \frac{x^5}{2\cdot 5} + \frac{x^7}{3\cdot 7}\right]_0^{0.4}$

≈ 0.020387

$\int_0^{0.4} \ln(1+x^2)\,dx \approx$ 0.020387

16. The function $f(x,y)$ has the first partial derivatives shown below. Classify all of the extrema (local minima/maxima and saddle point(s) of $f(x,y)$.

$x = y$

$f_x(x,y) = x - y = 0 \quad f_y(x,y) = y^3 - x = 0$

$y^3 - y = 0$

$y(y^2-1) = 0$

$y = \pm 1, 0$

(A) There are two saddle points and one local minimum

(B) There are two local maxima and one local minimum

(C) There are three saddle points

(D) There are two local minima and one saddle point

(E) There is one saddle point, one local maximum, and one local minimum

(F) There are two saddle points and one local maximum

	$f_{xx} = 1$	$f_{yy} = 3y^2$	$f_{xy} = -1$	D
(0,0)	1	0	−1	$0-(-1)^2 = -1$ → saddle
(1,1)	1	3	−1	$3-(-1)^2 = 2$ → rel min
(−1,−1)	1	3	−1	$3-(-1)^2 = 2$ → rel min

17. A bakery sells a particular cupcake where the customer satisfaction if given by $S(x, y) = 6x^{3/2}y$, where x is the grams of sugar and y is the grams of spice. If the sugar and spice used must satisfy $9x + y = 4$, what is the maximum customer satisfaction that can be achieved? Round your answer to 2 decimal places. (Note the function is only defined when $x \geq 0$ and $y \geq 0$)

$\to g$

$S_x = 6 \cdot \frac{3}{2} x^{1/2} y = 9x^{1/2} y$ $\qquad g_x = 9$

$S_y = 6x^{3/2}$ $\qquad g_y = 1$

$$\begin{cases} 9x^{1/2} y = 9\lambda & (1) \\ 6x^{3/2} = \lambda & (2) \\ 9x + y = 4 & (3) \end{cases}$$

Plug (2) into (1).

$9x^{1/2} y = 9 \cdot 6 x^{3/2}$

$9x^{1/2} y - 9 \cdot 6 x^{3/2} = 0$

$9x^{1/2}(y - 6x^{2/2}) = 0$

$9x^{1/2}(y - 6x) = 0$

$x = 0, \quad y = 6x$

$\downarrow \qquad \downarrow$

$S(x,y) = 0$ Nope

Plug into (3)

$9x + (6x) = 4$

$15x = 4$

$x = \frac{4}{15} \to y = 6 \cdot \frac{4}{15} = \frac{8}{5}$

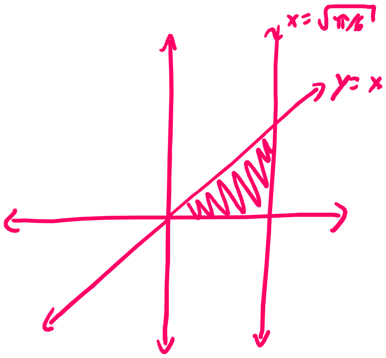
$S\left(\frac{4}{15}, \frac{8}{5}\right) \approx$

Maximum customer satisfaction = 1.322

18. Evaluate the definite integral

$$\int_0^{\sqrt{\pi/6}} \int_y^{\sqrt{\pi/6}} \cos(x^2)\, dx dy = \int_{x=0}^{x=\sqrt{\frac{\pi}{6}}} \int_{y=0}^{y=x} \cos(x^2)\, dy dx$$

(HINT: Change the order of integration.)



$$= \int_0^{\sqrt{\pi/6}} x \cos(x^2)\, dx$$

$u = x^2$
$du = 2x\,dx$ $\qquad \int \frac{1}{2} \cos(u)\, du$

$$= \frac{1}{2} \sin(u) = \frac{1}{2} \sin(x^2) \Big]_0^{\sqrt{\pi/6}}$$

$$= \frac{1}{2} \sin\left(\frac{\pi}{6}\right) - \frac{1}{2} \sin(0)$$

$\int_0^{\sqrt{\pi/6}} \int_y^{\sqrt{\pi/6}} \cos(x^2)\, dx dy =$ 1/4