Wednesday, September 17, 2025 10:27 AM

Basic Limit

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

Idea: $\lim_{x\to 0}\frac{\sin(\square)}{\square}=1$

Ex 1: $\lim_{x\to 0}\frac{\sin(4x)}{x}$

$$=\lim_{x\to 0}\frac{\sin(4x)}{4x}\cdot\frac{4x}{x}$$

$$=\left(\lim_{x\to 0}\frac{\sin(4x)}{4x}\right)\left(\lim_{x\to 0}\frac{4x}{x}\right)$$

$$=1$$

$$=1\cdot\left(\lim_{x\to 0}4\right)=4$$

Ex 2: $\lim_{x\to 0}\frac{\sin(3x)}{\sin(5x)}$

$$=\lim_{x\to 0}\frac{\sin(3x)}{3x}\cdot\frac{3x}{\sin(5x)}$$

$$=\left(\lim_{x\to 0}\frac{\sin(3x)}{3x}\right)\left(\lim_{x\to 0}\frac{3x}{\sin(5x)}\right)$$

$$=1$$

$$=1\cdot\lim_{x\to 0}\frac{5x}{\sin(5x)}\cdot\frac{3}{5}$$

$$=\left(\lim_{x\to 0}\frac{5x}{\sin(5x)}\right)\left(\lim_{x\to 0}\frac{3}{5}\right)$$

$$=1$$

$$=\cdot$$

$\lim_{x\to 0}\frac{\sin(\square)}{\square}=1=\frac{1}{1}$

$\lim_{x\to 0}\frac{\square}{\sin(\square)}=1$

$= 3/5$

Derivatives of trig functions

$$\frac{d}{dx}(\sin(x)) = \cos(x)$$

$$\frac{d}{dx}(\cos(x)) = -\sin(x)$$

$$\frac{d}{dx}(\tan(x)) = \sec^2(x)$$

$$\frac{d}{dx}(\cot(x)) = -\csc^2(x)$$

$$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$$

$$\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$$

Ex 3: Find y' of

$y = 3\sin(x) - 4\cos(x)$

$y' = 3\frac{d}{dx}(\sin(x)) - 4\frac{d}{dx}(\cos(x))$

$= 3\cos(x) - 4(-\sin(x))$

$= 3\cos(x) + 4\sin(x)$

Ex 4: Given $h(x) = x^2\sin(x)$. Compute $h'(\pi/6)$

$u(x) = x^2$ $\quad$ $v(x) = \sin(x)$

$u'(x) = 2x$ $\quad$ $v'(x) = \cos(x)$

By product rule,

$h'(x) = u'(x)v(x) + u(x)v'(x)$

$= 2x\sin(x) + x^2\cos(x)$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$$h'\left(\frac{\pi}{6}\right) = 2\left(\frac{\pi}{6}\right)\left(\frac{1}{2}\right) + \left(\frac{\pi}{6}\right)^2\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\pi}{6} + \frac{\pi^2\sqrt{3}}{36\cdot 2} = \frac{\pi}{6} + \frac{\pi^2\sqrt{3}}{72}$$

Ex 5: Let $h(x) = \dfrac{\cos(x)}{x+\cos(x)}$. Find $h'(x)$

Ex 3: Let $h(x) = \dfrac{\cos(x)}{x+\cos(x)}$. Find $h'(x)$

$u(x) = \cos(x)$ $\quad v(x) = x + \cos(x)$

$u'(x) = -\sin(x)$ $\quad v'(x) = 1 - \sin(x)$

By quotient rule, $h'(x) = \dfrac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}$

$$h'(x) = \frac{-\sin(x)[x+\cos(x)] - \cos(x)[1-\sin(x)]}{(x+\cos(x))^2}$$

$$= \frac{-x\sin(x) - \sin(x)\cos(x) - \cos(x) + \cos(x)\sin(x)}{(x+\cos(x))^2}$$

$$= \frac{-x\sin(x) - \cos(x)}{(x+\cos(x))^2}$$

Blooket | Question Set - Answer Key

Name _____
Date _____
Class _____

Derivative Rules

Everything before Chain Rule and Derivative of Exp, Log, and Inverse Trig.

1. Find the derivative

$\frac{d}{dx}(2023)$

a) 2023
b) 1
c) 0 ✔
d) -2023

2. Find the derivative

$\frac{d}{dx}(\pi^2)$

a) 2π
b) 1
c) 0 ✔
d) -2π

3. Find the derivative

$\frac{d}{dx}(3x^3)$

a) $3x^2$
b) $9x^2$ ✔
c) x^2
d) $9x^3$

4. Find the derivative

$\frac{d}{dx}\left(-\frac{1}{4}x^2\right)$

a) $-\frac{1}{2}x$ ✔
b) $\frac{1}{2}x$
c) $-\frac{1}{2}x^2$
d) $\frac{1}{2}x^2$

5. Find the derivative

$$\frac{d}{dx}\left(-2x^4+\frac{x^2}{2}-x\right)$$

a) $8x^3+x-1$

b) $-8x^3+x$

c) $-2x^3+x-1$

d) $-8x^3+x-1$ ✔

6. Find the derivative

$$\frac{d}{dx}\left(\sin x\right)$$

a) $\sin x$

b) $-\sin x$

c) $\cos x$ ✔

d) $-\cos x$

7. Find the derivative

$$\frac{d}{dx}\left(\cos x\right)$$

a) $\sin x$

b) $-\sin x$ ✔

c) $\cos x$

d) $-\cos x$

8. Find the derivative

$$\frac{d}{dx}\left(\sec x\right)$$

a) $\sec x\tan x$ ✔

b) $-\sec x\tan x$

c) $\sec^2 x$

d) $-\sec^2 x$

9. Find the derivative

$$\frac{d}{dx}\left(\tan x\right)$$

a) $\sec x\tan x$

b) $-\sec x\tan x$

c) $\sec^2 x$ ✔

d) $-\sec^2 x$

10. Find the derivative

$\frac{d}{dx}\left(\cot x\right)$

a) $\csc x \cot x$

b) $-\csc x \cot x$

c) $\csc^2 x$

d) $-\csc^2 x$ ✔

11. Find the derivative

$\frac{d}{dx}\left(\csc x\right)$

a) $\csc x \cot x$

b) $-\csc x \cot x$ ✔

c) $\csc^2 x$

d) $-\csc^2 x$

12. Find the derivative

$\frac{d}{dx}\left(\frac{x^2+x^4}{x^3}\right)$

a) $-x^{-2}+1$ ✔

b) $-x^{-2}+x$

c) $-x^{-1}+1$

d) $x^{-2}+1$

13. Find the derivative

$\frac{d}{dx}\left(\frac{2x+1}{2x-1}\right)$

a) $\frac{4}{(2x-1)^2}$

b) $\frac{-4}{(2x-1)^2}$ ✔

c) $\frac{8x}{(2x-1)^2}$

d) $\frac{-8x}{(2x-1)^2}$

14. Find the derivative

$\frac{d}{dx}\left(\left(x^2+1\right)\left(x^2-1\right)\right)$

a) $4x^3$ ✔

b) $-4x^3$

c) $4x^3-1$

d) $4x^3+1$

15. Find the derivative

$\frac{d}{dx}\left(\sqrt{x}\right)$

a) $\frac{1}{2}x^{-\frac{3}{2}}$

b) $\frac{1}{2}x^{-1}$

c) $-\frac{1}{2}x^{-\frac{1}{2}}$

d) $\frac{1}{2}x^{-\frac{1}{2}}$ ✔

16. Find the derivative

$\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)$

a) $\frac{1}{2}x^{-\frac{3}{2}}$

b) $-\frac{1}{2}x^{-\frac{1}{2}}$

c) $-\frac{1}{2}x^{-\frac{3}{2}}$ ✔

d) $\frac{1}{2}x^{-\frac{1}{2}}$

17. Find the derivative

$\frac{d}{dx}\left(x\sin x\right)$

a) $\cos x - x\sin x$

b) $\sin x + x\cos x$ ✔

c) $\cos x + x\sin x$

d) $\sin x - x\cos x$

18. Find the derivative

$\frac{d}{dx}\left(x\cos x\right)$

a) $\cos x - x\sin x$ ✔

b) $\sin x + x\cos x$

c) $\cos x + x\sin x$

d) $\sin x - x\cos x$

19. Find the derivative

$\frac{d}{dx}\left(x^2\sin x\right)$

a) $2x\cos x + x^2\sin x$

b) $2x\sin x - x^2\cos x$

c) $2x\cos x - x^2\sin x$

d) $2x\sin x + x^2\cos x$ ✔

20. Find the derivative

$\frac{d}{dx}\left(x^2 \cos x\right)$

a) $2x \cos x + x^2 \sin x$

b) $2x \sin x - x^2 \cos x$

c) $2x \cos x - x^2 \sin x$ ✔

d) $2x \sin x + x^2 \cos x$

21. Find the derivative

$\frac{d}{dx}\left(5 \sin x\right)$

a) $-5 \cos x$

b) $5 \cos x$ ✔

c) $5 \sin x$

d) $0 + \cos x$

22. Find the derivative

$\frac{d}{dx}\left(6 + \cos x\right)$

a) $-\sin x$ ✔

b) $\sin x$

c) $-6 \cos x$

d) $-6 \sin x$

23. Find the derivative

$\frac{d}{dx}\left(\frac{\sin x}{x}\right)$

a) $\frac{x \cos x - \sin x}{x^2}$ ✔

b) $\frac{-x \sin x - \cos x}{x^2}$

c) $\frac{-x \cos x + \sin x}{x^2}$

d) $\frac{x \sin x - \cos x}{x^2}$

24. Find the derivative

$\frac{d}{dx}\left(\frac{\cos x}{x}\right)$

a) $\frac{x \cos x - \sin x}{x^2}$

b) $\frac{-x \sin x - \cos x}{x^2}$ ✔

c) $\frac{-x \cos x + \sin x}{x^2}$

d) $\frac{x \sin x - \cos x}{x^2}$

25. Find the derivative

$\frac{d}{dx}\left(\sin x \cos x\right)$

a) 1

b) $\sin x + \sin x \cos x$

c) $\cos^2 x - \sin^2 x$ ✔

d) $\cos^2 x + \sin^2 x$

26. Find the derivative

$\frac{d}{dx}\left(\sqrt[3]{x}\right)$

a) $\frac{1}{3}x^{-\frac{2}{3}}$ ✔

b) $\frac{1}{3}x^{-2}$

c) $\frac{1}{3}x^{-\frac{1}{2}}$

d) $\frac{1}{2}x^{-\frac{1}{2}}$

27. Find the derivative

$\frac{d}{dx}\left(\frac{7}{x^5}\right)$

a) $-35x^{-4}$

b) $-35x^{-6}$ ✔

c) $35x^4$

d) $7x^{-6}$

28. Find the derivative

$\frac{d}{dx}\left(\frac{x}{2\sin x}\right)$

a) $\frac{2\sin x + 2x\cos x}{4\sin^2 x}$

b) $\frac{\sin x - x\cos x}{4\sin^2 x}$

c) $\frac{2\sin x - 2x\cos x}{4\sin^2 x}$ ✔

d) $\frac{2\sin x - 2x\cos x}{2\sin^2 x}$

29. Find the derivative

$\frac{d}{dx}\left(\frac{x}{3\cos x}\right)$

a) $\frac{\cos x + x\sin x}{9\cos^2 x}$

b) $\frac{3\cos x - 3x\sin x}{9\cos^2 x}$

c) $\frac{3\cos x + 3x\sin x}{3\cos^2 x}$

d) $\frac{3\cos x + 3x\sin x}{9\cos^2 x}$ ✔

30. Find the derivative

$$\frac{d}{dx}\left(\pi x\right)$$

a) π ✔

b) 0

c) $-\pi$

d) 1