

Problems to know (Some solutions below)

pg. 2 Problem (iv) $e^{2x^2 - 7x - 15} = 1$ Answer: $x = 5, -3/2$

Problem (b)(i) $\frac{\log_{15}(9) + \log_{15}(25)}{e^{3\ln(5) + 2\ln(7)}}$ Answer: $\frac{2}{5^3 \cdot 7^2}$

pg 4 Problem (v) $\sin^{-1}(\cos(\frac{3\pi}{5}))$ Answer: $-\pi/10$

Problem (vi) $\sin^{-1}(\cos(\frac{4\pi}{7}))$ Answer: $-\pi/14$

pg 8 Problem (ii) $\sin(x) = \cos(2x)$ Answer: $\pi/6, 5\pi/6, 3\pi/2$

Problem (iii) $-\sin^2(x) = \cos(2x)$ Answer: $\frac{\pi}{2}, \frac{3\pi}{2}$

Recall composition of functions.

Let $y = f(g(x)) \Rightarrow$
 g - inner function
 f - outer function

Ex 1: Determine f and g for $y = (3x+1)^2$

$$f(x) = x^2$$

$$g(x) = 3x+1$$

Check $y = f(g(x)) = f(3x+1) = (3x+1)^2 \checkmark$

Ex 2: Determine f and g for $y = \sin^2 x = (\sin x)^2$

$$f(x) = x^2$$

$$g(x) = \sin x$$

Check $y = f(g(x))$

Ex 3: Determine f and g for $y = \ln(2x)$

$$f(x) = \ln(x)$$

$$f(x) = \ln(x)$$

$$g(x) = 2x$$

Check $y = f(g(x))$

Ex 4: Determine f and g for $y = 3\sqrt{2x+1}$

$$f(x) = 3\sqrt{x}$$

$$g(x) = 2x+1$$

Check $y = f(g(x))$

Chain Rule

Let $y = f(g(x))$

Version 1: $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$ *

Version 2: $\frac{dy}{dx} = \frac{d}{dx}[f(\underbrace{g(x)}_u)] = \frac{df}{du} \cdot \frac{du}{dx}$

Ex 1: Find y' of $y = (3x+1)^2$

Expand $y = (3x)^2 + 2(3x)(1) + 1^2$

$$= 9x^2 + 6x + 1$$

$$y' = 9(2)x' + 6 + 0 = 18x + 6$$

Ex 2: Find y' of $y = 2\cos^3 x$

Rewrite $y = 2(\cos(x))^3$

$$f = 2x^3 \quad g = \cos(x)$$

$$f' = 6x^2 \quad g' = -\sin(x)$$

By Chain Rule, $y' = f'(g(x)) \cdot g'(x)$

By Chain rule:

$$\begin{aligned}
 y' &= f'(g(x)) \cdot g'(x) \\
 &= f'(\cos(x)) \cdot (-\sin(x)) \\
 &= 6(\cos(x))^2(-\sin(x)) \\
 &= -6\cos^2(x)\sin(x)
 \end{aligned}$$

Ex 3: Find y' of $y = \left(\frac{2x}{3x^2+x}\right)^3$

Simplify and rewrite y .

$$\begin{aligned}
 y &= \left(\frac{\cancel{2x}}{\cancel{x}(3x+1)}\right)^3 \\
 &= \left(\frac{2}{3x+1}\right)^3 \\
 &= \frac{2^3}{(3x+1)^3} \\
 &= \frac{8}{(3x+1)^3} \\
 &= 8(3x+1)^{-3}
 \end{aligned}$$

Find y' .

$$\begin{aligned}
 f(x) &= 8x^{-3} & g(x) &= 3x+1 \\
 f' &= 8(-3)x^{-4} & g' &= 3 \\
 &= -24x^{-4}
 \end{aligned}$$

By Chain Rule

$$\begin{aligned}
 y' &= f'(g(x)) \cdot g'(x) \\
 &= f'(3x+1) \cdot 3 \\
 &= -24(3x+1)^{-4} \cdot 3 \\
 &= \frac{-72}{(3x+1)^4}
 \end{aligned}$$

pg. 2 Problem (iv) $e^{2x^2 - 7x - 15} = 1$

Problem (b)(i) $\frac{\log_{15}(9) + \log_{15}(25)}{e^{3\ln(5) + 2\ln(7)}} = \frac{\log_{15}(\overbrace{9 \cdot 25}^{(3 \cdot 5)^2})}{e^{\ln(5^3) + \ln(7^2)}} = \frac{2}{5 \cdot 7^2}$

pg 4 Problem (v) $\sin^{-1}\left(\cos\left(\frac{3\pi}{5}\right)\right)$
 Problem (vi) $\sin^{-1}\left(\cos\left(\frac{4\pi}{7}\right)\right)$

$\cos(x) = \sin\left(\frac{\pi}{2} - x\right)$

(v) $\sin^{-1}\left(\cos\left(\frac{3\pi}{5}\right)\right)$

$$\begin{aligned}
 & (v) \sin \left(\cos \left(\frac{\pi}{5} \right) \right) \\
 & \quad \quad \quad \sin \left(\frac{\pi}{2} - \frac{3\pi}{5} \right) = \sin \left(\frac{5\pi}{10} - \frac{6\pi}{10} \right) = \sin \left(-\frac{\pi}{10} \right) \\
 & \quad \quad \quad = \sin^{-1} \left(\sin \left(-\frac{\pi}{10} \right) \right) = -\frac{\pi}{10}
 \end{aligned}$$

pg 8 Problem (ii) $\sin(x) = \cos(2x)$ } $\cos(2x) = 1 - 2\sin^2(x)$
 Problem (iii) $-\sin^2(x) = \cos(2x)$ } This problems are $[0, 2\pi)$

$$\begin{aligned}
 & (ii) \sin(x) = 1 - 2\sin^2(x) \\
 & \quad 2\sin^2(x) + \sin(x) - 1 = 0 \\
 & \quad u = \sin(x) \\
 & \quad 2u^2 + u - 1 = 0 \\
 & \quad 2u^2 + 2u - u - 1 = 0 \\
 & \quad 2u(u+1) - (u+1) = 0 \\
 & \quad (2u-1)(u+1) = 0 \\
 & \quad u = \frac{1}{2} \quad u = -1 \\
 & \quad \sin \theta = \frac{1}{2} \quad \sin \theta = -1 \\
 & \quad \theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad \theta = \frac{3\pi}{2}
 \end{aligned}$$

$$\begin{aligned}
 & (iii) -\sin^2(x) = 1 - 2\sin^2(x) \\
 & \quad \sin^2(x) = 1 \\
 & \quad \sin(x) = \pm 1 \\
 & \quad x = \frac{\pi}{2}, \frac{3\pi}{2}
 \end{aligned}$$