Lesson 21: What derivatives tell us (Part 2)

Monday, October 20, 2025 10:26 AM

Study Guide Questions to <u>KNOW</u>

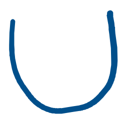
1.2, 1.4.8, 1.4.9, 1.4.10, 2.1.4, 2.2, 3.2, 3.3, 4.3, 5.2, 5.3, 6.1.4, 6.3, 7.2.2, 7.2.3, 7.4, 7.6, 7.7, 7.9, 7.10, 7.14

<u>Concavity of a Function</u>

Suppose $f''(x)$ exists on an open interval, I. Then

(a) If $f''(x) > 0$ for all $x \in I$ (in), then $f(x)$ is concave up on I.

(b) If $f''(x) < 0$ for all $x \in I$, then $f(x)$ is concave down on I.



Concave up like a cup

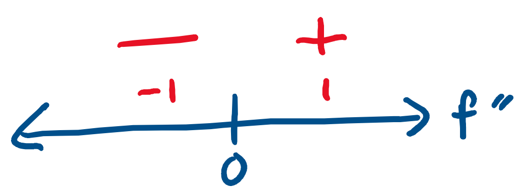
Concave down like frown

<u>Ex 1:</u> Determine the largest open interval(s) on which $f(x) = x^3 - x$ is concave up or down.

$f'(x) = 3x^2 - 1$

$f''(x) = 6x = 0 \Rightarrow x = 0$

Next draw a # line w/ the x-values found when $f''(x) = 0$.



$f''(-1) = 6(-1) = -$

$f''(1) = 6(1) = +$

Concave Up: $(0, \infty)$ Concave down: $(-\infty, 0)$

<u>Inflections Pts:</u> Pts where concavity changes.

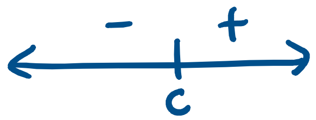
① Find the pts on the curve where $f''(x) = 0$ or DNE.

Inflections pts: Pts where concavity changes,

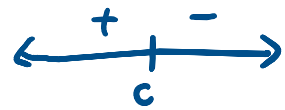
① Find the pts on the curve where $f''(x) = 0$ or DNE.

② Test whether the concavity changes at these pts.

i.e.



f'' or



where c is a pt found in ①.

Ex 2: Find the inflection pt(s) of $f(x) = \frac{3}{5}x^5 - x^4$ if they exists,

Step 1: Find $f''(x) = 0$.

$$f'(x) = \frac{3}{5}(5)x^4 - 4x^3$$

$$= 3x^4 - 4x^3$$

$$f''(x) = 3(4)x^3 - 4(3)x^2$$

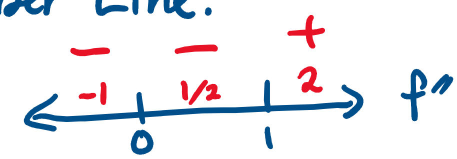
$$= 12x^3 - 12x^2 = 0$$

$$12x^2(x-1) = 0$$

$$12x^2 = 0 \qquad x - 1 = 0$$

$$x = 0 \qquad x = 1$$

Step 2: Number Line.



$f''(x) = 12x^2(x-1)$

$f''(-1) = 12(-1)^2(-1-1) = + \cdot -$

$f''(\frac{1}{2}) = 12(\frac{1}{2})^2(\frac{1}{2}-1) = + \cdot -$

$f''(2) = 12(2)^2(2-1) = + \cdot +$

By def, inflection pt only at $x = 1$.

How to figure out the shape of a graph

How to figure out the shape of a graph

Ex 3: Let $y = 3x^4 - 4x^3 - 6x^2 + 12x + 1$

$$y' = 12x^3 - 12x^2 - 12x + 12$$

Find when $y' = 0$.

$$12x^3 - 12x^2 - 12x + 12 = 0$$

$$12(\underbrace{x^3 - x^2}\underbrace{- x + 1}) = 0$$

$$12[x^2(x-1) - 1(x-1)] = 0$$

$$12[x^2 - 1](x-1) = 0$$

$$12(x-1)(x+1)(x-1) = 0$$

$$x = 1, -1$$

$y' = 0$
@ $x = -1, 1$

Find when $y'' = 0$.

$$y' = 12x^3 - 12x^2 - 12x + 12$$

$$y'' = 36x^2 - 24x - 12 = 0$$

$$= 12[3x^2 - 2x - 1] = 0$$

$$12[3x^2 + x - 3x - 1] = 0$$

$$12[x(3x+1) - 1(3x+1)] = 0$$

$$12(x-1)(3x+1) = 0$$

$x - 1 = 0$ $\quad$ $3x + 1 = 0$

$x = 1$ $\quad$ $x = -1/3$

-3
$\wedge$
$1 - 3$

$y'' = 0$
@ $x = 1, -\frac{1}{3}$

Create a Table.

X		-1		$-1/3$		1	
$f'(x)$	$-$ -2	0	$+$ $-2/3$	$+$	$+$	0	$+$ 2
$f''(x)$	$+$	$+$	$+$	0	$-$	0	$+$ 2

$f' = 12(x-1)^2(x+1) = 0$
@ $x = 1, -1$

$f'(x)$	-2	0	-2/3				2
$f''(x)$	+ -2	+	+	0	- 0	0	+ 2
Conclusion		local min $\overset{-}{\longleftarrow}\!\!\!\!\!\!\mid\!\!\!\!\!\!\overset{+}{\longrightarrow} f'$ -		$\overset{+ \quad -}{\longleftrightarrow} f''$ inf pt		$\overset{- \quad +}{\longleftrightarrow} f''$ inf pt	

$f''(x) = 12(x-1)(3x+1)$

@ $x = 1, -1/3$

$f''(-2) = 12(-2-1)(3(-2)+1)$

$= + \cdot - \cdot -$

$f''(0) = 12(-1)(1)$

$$\underset{-1}{\overset{-}{\longleftarrow}}\!\!\mid\!\!\overset{+}{\underset{}{-2/3}}\!\!\!\!\underset{1}{\mid}\!\!\longrightarrow f'$$