MA 165 LESSONS 23+24: OPTIMIZATION

Optimization Problems: Often this includes finding the maximum or minimum value of some function.

- e.g. The minimum time to make a certain journey,
- e.g. The minimum cost for constructing some object,
- e.g. The maximum profit to gain for a business, and so on.

How do we solve an optimization problem?

- Determine a function (**known as objective function)** that we need to maximize or minimize.

- Determine if there are some constraints on the variables. (The equations that describe the constraints are called **the constraint equations**.)
    - If there are constraint equations, rewrite the **objective function** as a function of only one variable.

- Then we can solve for absolute maximum or minimum like we did before.
    - Using either the First Derivative Test or Second Derivative Test.

Recipe for Solving an Optimization Problem

Step 1: Identify what quantity you are trying to optimize.

Step 2: Draw a picture (if applicable), corresponding to the problem, and label it with your variables.

Step 3: Express the variable to be optimized as a function of the variables you used in Step 2.

Step 4: Find relations among the variables from Step 2 and express the variable to be optimized a function of just one of the variables from Step 2.

Step 5: Identify the domain for the function you found in Step 4.

Step 6: Find the absolute extrema of the variable to be optimized on this domain.

Step 7: Reread the question and be sure you have answered exactly what was asked.

Example 1: A carpenter is building a rectangular room with a fixed perimeter of 54 feet. What are the dimensions of the largest room that can be built? What is its area?

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Example 2: A rancher plans to make four identical and adjacent rectangular pens against a barn, each with an area of 25 m^2. What are the dimensions of each pen that minimize the amount of fence that must be used?

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Example 3: An open-top box with a square base is to have a volume of 8 cubic feet. Find the dimensions of the box that can be made with the least amount of material.

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<u>Example 4:</u> Find the point on the graph of $f(x) = 2x + 4$ that is the closest to the point $(1,3)$.

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<u>Example 5:</u> A piece of wire of length 40 is cut, and the resulting two pieces are formed to make a circle and a square. Where should the wire be cut to minimize the combined area of the circle and the square?

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<u>Example 6:</u> From a thin piece of cardboard 20 in by 20 in, square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume?

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