Consider the equation $F'(x) = f(x)$

Two ways to interpret this:

① $f(x)$ = derivative of $F(x)$

② $F(x)$ = antiderivative

Notation: $F(x) = \int f(x)dx$

With antiderivatives start with $f(x)$ and find $F(x)$.

Ex 1: ⓐ Differentiate $F(x) = x^2 + 2$. $F'(x) = 2x$

ⓑ Find $\int 2x\,dx$

What function $F(x)$ has $2x$ as its derivative?

By ⓐ, one such $F(x)$ is $x^2 + 2$.

But so are:

- x^2
- $x^2 - 1234$
- $x^2 + (\text{constant})$

Why? Derivative of a constant is zero.

To account for this, use C as an arbitrary constant

$$\int 2x\,dx = x^2 + C$$

Process of finding all the antiderivatives of a function is called indefinite integration.

Denoted by $\underbrace{\int f(x)dx}_{\text{Read as "integral of } f(x)\text{"}} = F(x) + C$ where C is a constant

- $\int$ integral sign
- $f(x)$ integrand
- x integration
- C constant of integration

Differentiation Rule	Integration Rules
$\frac{d}{dx}(C) = 0$	$\int 0\,dx = C$
$\frac{d}{dx}(kx) = k$	$\int k\,dx = kx + C$

$\frac{d}{dx}(kx) = k$	$\int k\,dx = kx + C$
$\frac{d}{dx}(k f(x)) = k f'(x)$	$\int k f'(x)\,dx = k \int f'(x)\,dx = k f(x) + C$
$\frac{d}{dx}(x^n) = n x^{n-1}$ $\frac{d}{dx}(x^{n+1}) = (n+1)x^n$	$\int n x^{n-1}\,dx = x^n + C$ $\int (n+1)x^n\,dx = x^{n+1} + C$ $(n+1)\int x^n\,dx = x^{n+1} + C$ $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

Ex 2: Find the indefinite integral

$$\int (x^2 + 2\sqrt{x})\,dx$$

$$\int (x^2 + 2x^{1/2})\,dx = \frac{x^{2+1}}{2+1} + 2\,\frac{x^{1/2+1}}{1/2+1} + C$$

$$= \frac{x^3}{3} + 2\,\frac{x^{3/2}}{3/2} + C$$

$$= \frac{x^3}{3} + 2 \cdot \frac{2}{3} x^{3/2} + C$$

$$= \frac{x^3}{3} + \frac{4}{3} x^{3/2} + C$$

Ex 3: Find the indefinite integral

$$\int (x^{-2} + x^2)\,dx$$

$$= \frac{x^{-2+1}}{-2+1} + \frac{x^{2+1}}{2+1} + C$$

$$= \frac{x^{-1}}{-1} + \frac{x^3}{3} + C$$

$$= -\frac{1}{x} + \frac{x^3}{3} + C$$

Differentiation Rule	Integration Rule
$\frac{d}{dx}(\sin(x)) = \cos(x)$	$\int \cos(x)\,dx = \sin(x) + C$
$\frac{d}{dx}(\cos(x)) = -\sin(x)$	$\int \sin(x)\,dx = -\cos(x) + C$
$\frac{d}{dx}(\tan(x)) = \sec^2(x)$	$\int \sec^2(x)\,dx = \tan(x) + C$
$\frac{d}{dx}(\cot(x)) = -\csc^2(x)$	$\int \csc^2(x)\,dx = -\cot(x) + C$

$\frac{d}{dx}(\cot(x)) = -\csc^2(x)$	$\int \csc^2(x)$
$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$	$\int \sec(x)\tan(x)\,dx = \sec(x) + C$
$\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$	$\int \csc(x)\cot(x)\,dx = -\csc(x) + C$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$ for $x>0$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$

ALWAYS take the derivative of your answer to check you are right, especially when you have trig functions.

Ex 4: Evaluate $\int \frac{\sin(x)+\cos(x)}{2}\,dx$

$= \frac{1}{2}\int (\sin(x)+\cos(x))\,dx$

$= \frac{1}{2}\Big[\underbrace{\int \sin(x)\,dx}_{-\cos x} + \underbrace{\int \cos(x)\,dx}_{\sin x}\Big]$

$= \frac{1}{2}(-\cos(x)+\sin(x)) + C$

Let's check our answer.

$\left(\frac{1}{2}(-\cos x+\sin x)\right)'$

$= \frac{1}{2}(-(-\sin x) + \cos x)$

$= \frac{\sin(x)+\cos(x)}{2}$ ✓

Ex 5: Evaluate $\int \left(x^e + \frac{1}{x} + 1\right)dx$

$= \int x^e\,dx + \int \frac{1}{x}\,dx + \int dx$

$= \frac{x^{e+1}}{e+1} + \ln\lvert x\rvert + x + C$

Let's our answer.

$\frac{d}{dx}\left(\frac{x^{e+1}}{e+1} + \ln\lvert x\rvert + x + C\right)'$

$= \frac{(e+1)x^e}{(e+1)} + \frac{1}{x} + 1$

$= x^e + \frac{1}{x} + 1$

How do I find C? Problems that involve us to find C are called Initial Value Problems.

Initial Value Problems.

<u>Initial Value Problem</u> is a differential eqn [eqn w/ derivatives] with an initial condition.

<u>Ex 6</u>: Determine $f(x)$ when $f'(x) = x^2 - 2x$ and $\underbrace{f(1) = 1/3}_{\text{Initial condition}}$.

First find $f(x) = \int f'(x)\,dx$

$$= \int x^2 - 2x\,dx$$

$$= \frac{x^3}{3} - \frac{2x^2}{2} + C$$

$$= \frac{x^3}{3} - x^2 + C$$

Now to find C we use $f(1) = 1/3$

$$\cancel{\tfrac{1}{3}} = f(1) = \cancel{\tfrac{1}{3}} - 1 + C$$

$$0 = -1 + C$$

$$C = 1$$

So $f(x) = \dfrac{x^3}{3} - x^2 + 1$