Section 1.4: Trigonometric Functions and Their Inverses

Def: Trigonometric Functions

$\sin\theta = \dfrac{opp}{hyp}$ $\cos\theta = \dfrac{adj}{hyp}$ $\tan\theta = \dfrac{opp}{\cancel{opp}\ adj}$

$\csc\theta = \dfrac{hyp}{opp}$ $\sec\theta = \dfrac{hyp}{adj}$ $\cot\theta = \dfrac{adj}{opp}$

Values of Sine, Cosine in the first quadrant

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	$0 = \sqrt{0}/2$	$\frac{1}{2} = \sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2 = 1$
cos	$1 = \sqrt{4}/2$	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	$\sqrt{0}/2 = 0$

To determine the values in the other quadrants use

S	A
T	C

where A is all trig values are positive
S only sine values are positive
T only tangent values are positive
C only cosine values are positive

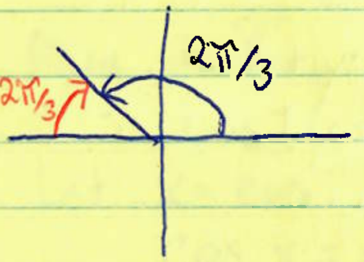
Ex: Evaluate $\sin(-\pi/3)$.

$-\pi/3$ is in Quadrant 4, so the value is negative.
The reference angle of $-\pi/3$ is $\pi/3$. So $\sin(\pi/3) = \sqrt{3}/2$
So the answer is $\boxed{-\sqrt{3}/2}$

Remember reference angles are angles that we know values of, and have the same "endpoint."

For example, if we have $2\pi/3$, the reference angle is $\pi/3$ as shown in the image to the left.

$\pi/3 = \pi - 2\pi/3$



Transformation of Trig Graphs

$$y = A\sin(B(\theta - C)) + D$$
$$y = A\cos(B(\theta - C)) + D$$

where $|A|$ is the amplitude,

$\frac{2\pi}{|B|}$ is the period,

C is the horizontal shift, and

D is the vertical shift

Inverse Trigonometric Functions

Def: $y = \sin^{-1}(x) \iff x = \sin(y)$ where $-\pi/2 \le y \le \pi/2$

$y = \cos^{-1}(x) \iff x = \cos(y)$ where $0 \le y \le \pi$.

The domain of both $\sin^{-1}(x)$ and $\cos^{-1}(x)$ is $(-1, 1)$

Ex: Evaluate

ⓐ $\sin^{-1}(\sqrt{3}/2)$

Let $x = \sin^{-1}(\sqrt{3}/2) \iff \sin(x) = \sqrt{3}/2$

This is true when $x = \pi/3$

ⓑ $\cos^{-1}(\cos(3\pi))$

BEWARE $\cos^{-1}$ and $\cos$ don't cancel out always.

First find $\cos(3\pi)$.

Remember 3π is the same as π. So

$$\cos(3\pi) = \cos(\pi) = -1$$

Plug -1 for $\cos(3\pi)$.

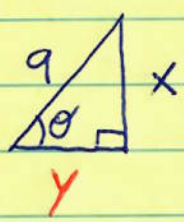
$$\cos^{-1}(\cos(3\pi)) = \cos^{-1}(-1)$$

Let $x = \cos^{-1}(-1)$

$\cos x = -1 \Rightarrow$ This happens at $x = \pi$.

So $\cos^{-1}(\cos(3\pi)) = \pi$ Not 3π

HW Question: (#22) Express θ in terms of x using the inverse sine, inverse tangent, and inverse secant functions.



First let's find all the sides. i.e. Find y. So use Pygatheorom Thm.

$$x^2 + y^2 = 9^2$$
$$x^2 + y^2 = 81$$
$$y^2 = 81 - x^2$$
$$y = \sqrt{81 - x^2}$$

(Positive cause its a length)

ⓐ $\theta = \sin^{-1}(\_\_\_\_)$

Remember this is the same as $\sin\theta = \_\_\_\_$

$\sin\theta = \frac{opp}{hyp} = \frac{x}{9}$ Answer: $\frac{x}{9}$

ⓑ $\theta = \tan^{-1}(\_\_\_\_)$

Remember this is the same as $\tan\theta = \_\_\_\_$

$\tan\theta = \frac{opp}{adj} = \frac{x}{y} = x/\sqrt{81-x^2}$ Answer: $x/\sqrt{81-x^2}$