9/5/25: Section 2.4 Infinite Limits

Recall from last class, we have 2 one-sided limits
$\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$

We can compute these limits numerically, graphically, and analytically. Last class these limits equal a # or DNE but can they equal something else? Yes

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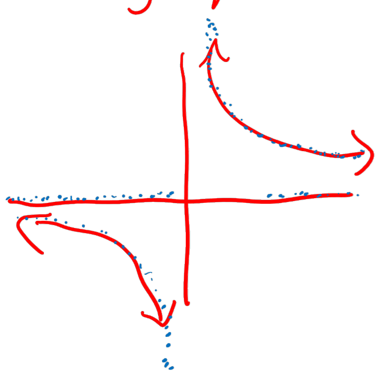
Ex 1: Let $f(x) = \frac{1}{x}$. Compute

Draw the graph



ⓐ $\lim_{x \to 0^-} \left(\frac{1}{x}\right) = -\infty$

ⓑ $\lim_{x \to 0^+} \left(\frac{1}{x}\right) = \infty$

ⓒ $\lim_{x \to 0} \left(\frac{1}{x}\right)$ DNE

b/c ⓐ $\neq$ ⓑ

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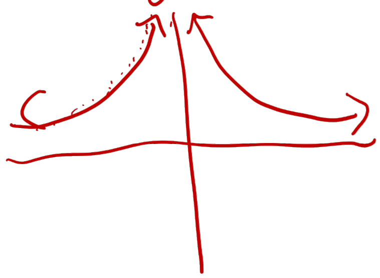
Ex 2: Let $f(x) = \frac{1}{x^2}$. Compute

(a) $\lim\limits_{x \to 0^-} \left(\frac{1}{x^2}\right) = \infty$

(b) $\lim\limits_{x \to 0^+} \left(\frac{1}{x^2}\right) = \infty$

(c) $\lim\limits_{x \to 0} \left(\frac{1}{x^2}\right) = \infty$

Draw the graph



If $f(x) = \frac{1}{x^n}$ for $n =$ odd # then it will act like Ex 1

$n =$ even # then it will act like Ex 2

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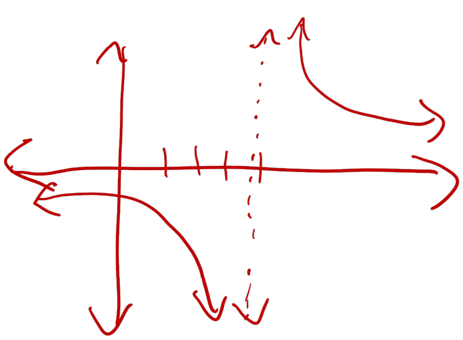
Ex: (a) $\lim\limits_{x \to 4^-} \frac{4}{x-4} = -\infty$

(b) $\lim\limits_{x \to 4^+} \frac{4}{x-4} = +\infty$

(c) $\lim\limits_{x \to 4} \frac{4}{x-4} = \text{DNE}$



Ex: (a) $\lim\limits_{x \to 4^-} \frac{-4}{x-4} = \infty$

(b) $\lim\limits_{x \to 4^+} \frac{-4}{x-4} = -\infty$

(c) $\lim\limits_{x \to 4} \frac{-4}{x-4} = \text{DNE}$

Be mindful of the negatives.

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But what if $f(x)=\frac{x^2-8x+15}{(x-2)^2}$ or $g(x)=\frac{x-13}{x^2-169}$?

Ex: $\lim_{x\to 2}\frac{x^2-8x+15}{(x-2)^2}$

Plug & chunch

$$\frac{2^2-8(2)+15}{(2-2)^2}=\frac{3}{0}\Rightarrow VA\ @\ x=2$$

First factor.

$$\frac{x^2-8x+15}{(x-2)^2}=\frac{(x-3)(x-5)}{(x-2)^2}$$

Now do the one-sided limit,

$$\lim_{x\to 2^-}\frac{(x-3)(x-5)}{(x-2)^2}$$

$$=\frac{(2^--3)(2^--5)}{(2^--2)^2}$$

$$=\frac{(-)(-)}{(-)^2}=\frac{+}{+}=+\infty$$

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$$\lim_{x\to 2^+}\frac{(x-3)(x-5)}{(x-2)^2}=\frac{(2^+-3)(2^+-5)}{(2^+-2)^2}=\frac{(-)(-)}{(+)^2}=+\infty$$

$$\lim_{x\to 2}\frac{x^2-8x+15}{(x-2)^2}=\infty$$

Section 2.5: Limits at ∞

So next we are doing $\lim_{x\to\infty} f(x)$ or $\lim_{x\to-\infty} f(x)$

These limits are nice b/c they tell you about endbehavior

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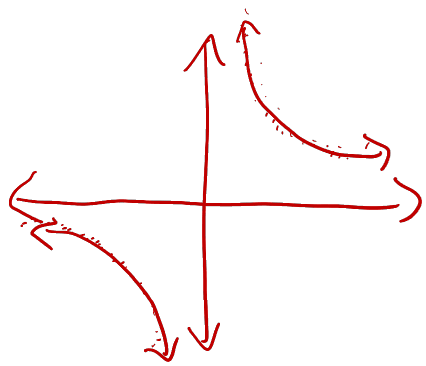
Ex: Let $f(x) = \frac{1}{x}$.



(a) $\lim_{x \to -\infty} \left(\frac{1}{x}\right) = 0$

(b) $\lim_{x \to \infty} \left(\frac{1}{x}\right) = 0$

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Ex: $\lim_{x \to \infty} \frac{8 + 8x + 9x^2}{x^2} \Rightarrow \lim_{x \to \infty} \frac{9\cancel{x^2}}{\cancel{x^2}} = 9$

Ex: $\lim_{x \to +\infty} \frac{x - 1}{\sqrt{x} - 1} \Rightarrow \lim_{x \to +\infty} \frac{x}{\sqrt{x}} = \lim_{x \to +\infty} x^{1/2} = \infty$

Ex: $\lim_{x \to \infty} \frac{3x^2 - 4x + 1}{x^4} \Rightarrow \lim_{x \to \infty} \frac{3x^2}{x^4} = \lim_{x \to \infty} \frac{3}{x^2} = 0$

$\lim_{x \to -\infty} \frac{3x^2 - 4x + 1}{x^4} = 0$

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