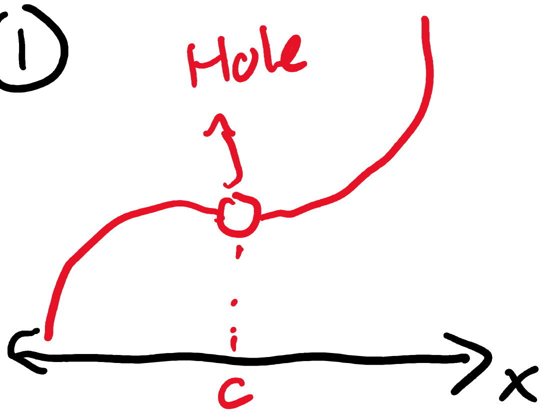
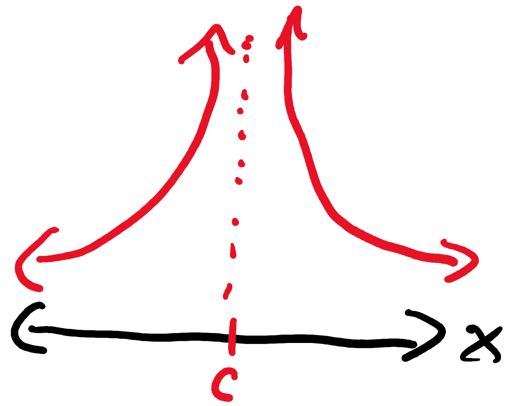
A function is continuous if there is no disruption in the graph.

The following 4 graphs show f(x) is discontinuous @ $x=c$

(1)

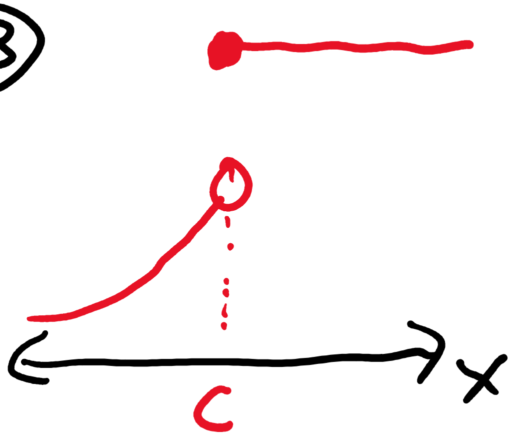


(2)

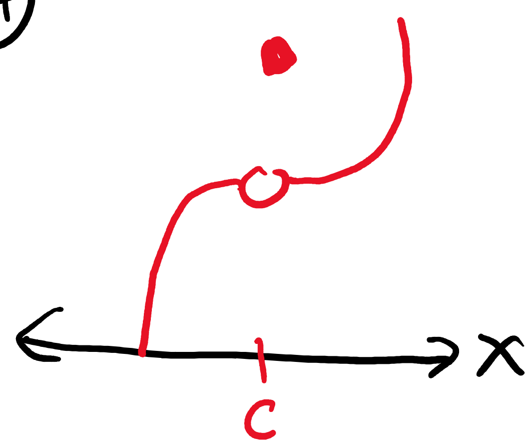


These 2 graphs have $f(c)$ undefined.

(3)



(4)



$f(c)$ is defined BUT $\lim_{x \to c} f(x) = DNE$

$f(c)$ is defined

$\lim_{x \to c} f(x)$ exists

BUT $\lim f(x) \neq f(c)$

<u>BUT</u> $\lim_{x \to c} f(x) \neq f(c)$

We can see a function $f(x)$ is continuous @ $x=c$ if the following is true:

1. $f(c)$ is defined (i.e. it has a value
2. $\lim_{x \to c} f(x)$ exists
3. $\boxed{\lim_{x \to c} f(x) = f(c)}$

If any of these 3 conditions aren't met, then we say $f(x)$ is discontinuous at $x=c$.

<u>Ex 1</u>: Discuss continuity of $f(x) = \dfrac{2x}{x^2 - x}$

When is $f(x)$ undefined?
(i.e. when is deno $= 0$)

$$x^2 - x = 0$$
$$x(x-1) = 0$$
$$x = 0, 1 \rightarrow \text{We have discontinuities @ } x = 0, 1$$

@ $x=0,$

But what kind of discontinuity are they?
Jump? Hole? VA?

Let's simplify to answer that question

$$f(x)=\frac{2x}{x^2-x}=\frac{2x}{x(x-1)}=\frac{2}{x-1}$$

B/c x canceled we have a hole @ $x=0$

What remains is VA $\Rightarrow$ $x=1$ is VA.

What factor cancels out? Hole
What remains?

Ex 2: Discuss continuity of

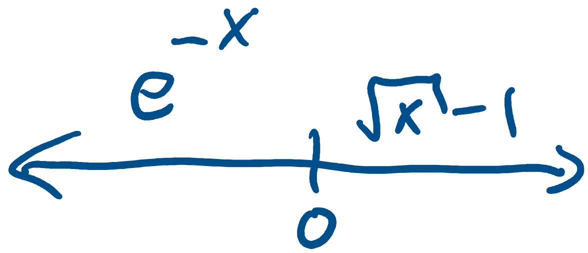
$$f(x)=\begin{cases} e^{-x} & \text{if } x\leq 0 \\ \sqrt{x}-1 & \text{if } x>0 \end{cases}$$

To determine continuity, we need to check

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=\lim_{x\to 0} f(x)=f(0)$$

For this question, we only need to check the left and right limits.

the left and right limits.



$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0} e^{-x} = e^{-0} = 1$$

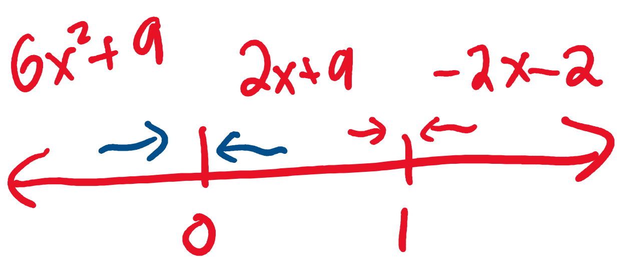
$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} (\sqrt{x}-1) = 0-1 = -1$$

Discontinuity @ $x=0$ $\Downarrow$ Jump

Ex 3: Discuss continuity of

$$f(x) = \begin{cases} 6x^2+9 & \text{if } x \le 0 \\ 2x+9 & \text{if } 0 < x < 1 \\ -2x-2 & \text{if } x \ge 1 \end{cases}$$

We may have discontinuities @ $x = 0, 1$



To check

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0} (6x^2+9) = 9$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} (2x+9) = 9$$

$\rightarrow$ No discontinuity @ $x=0$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0}$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1}(2x+9) = 11$$

$\neq$ Discontinuity @ $x=1$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1}(-2x-2) = -4$$

$\Downarrow$

Jump