Lesson 7: The Derivative

Wednesday, September 10, 2025 9:54 AM

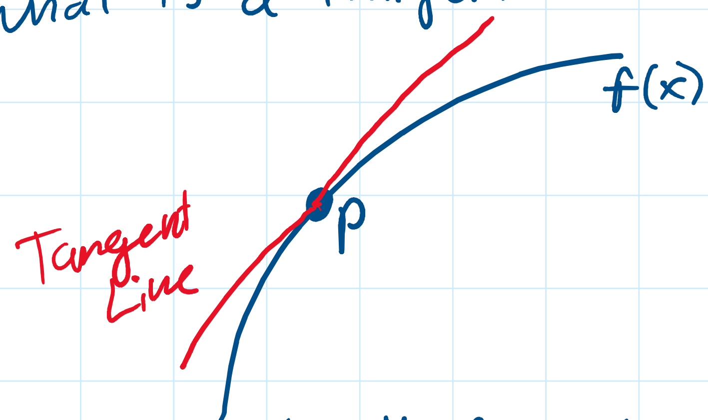
Annoucements

Should be

1. Study Guide is posted for Exam 1
2. Reminder Friday marks 2 weeks to Exam 1
   ↳ 10 questions, in class, no calculator :(
3. No physical class on Friday

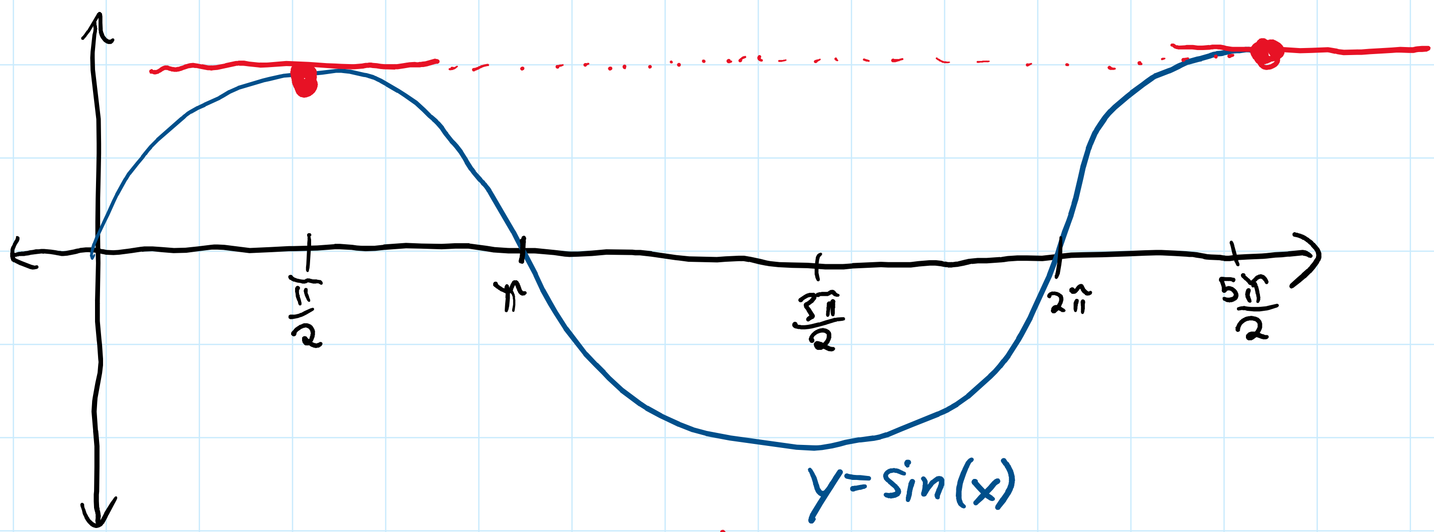
Tangent Lines are Important in Calculus!

What is a Tangent line?



Graph of f(x) at a point p is a straight line that touches f(x) at point p, but does not cross f(x)

Note that this "definition" doesn't work all the time

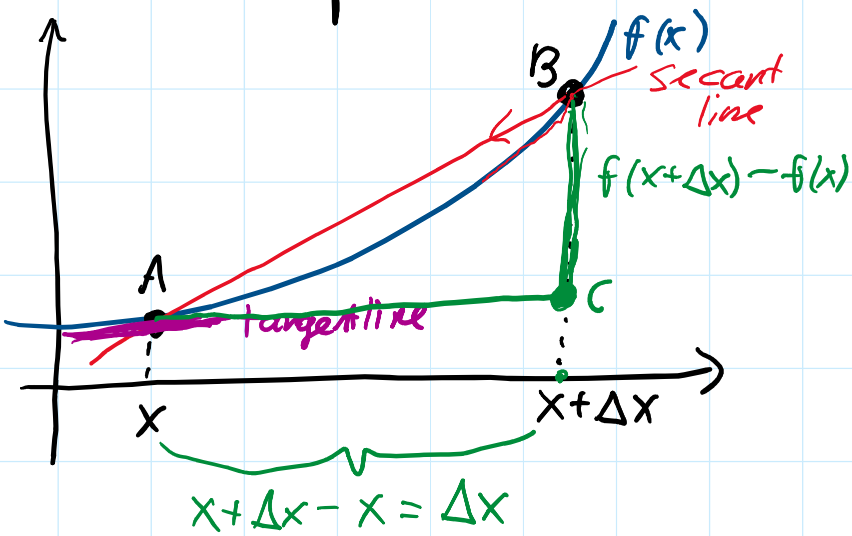


But the red lines cross!

We need a precise definition for ALL scenarios. Before we do so, let's recall secant lines.

We need a precise definition for ALL scenarios. Before we do so, let's recap secant lines.

A secant line of $f(x)$ is a straight line that goes through two distinct points on $f(x)$.



If we move B along the curve $f(x)$ towards A, then the secant line becomes tangent line.

Difference of Quotient

$$\text{Slope of secant line} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$\text{Slope of Tangent} = \lim_{\Delta x \to 0} \text{Slope of secant line} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

How is the slope of the tangent line related to the derivative? They are the same.

Def: The derivative of $f(x)$ at x, denoted by $f'(x)$, is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \quad \text{where } h = \Delta x$$

provided the limit exists.

Different Notations: y', $\frac{dy}{dx}$, $f'(x)$, $\frac{d}{dx}[f(x)]$

Alternate Def: $f'(x) = \lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

Game Plan: To find derivative using the limit definition, following steps:

① $f(x+h)$

② $-f(x)$

③ Find $f(x+h) - f(x)$ by add ① + ②

④ Find $\dfrac{f(x+h) - f(x)}{h}$ by dividing ③ by h.

⑤ Find $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ by taking the $\lim_{h \to 0}$ of 4.

Ex 1: Find the derivative of $f(x) = x + 5$ using limit def.

Step 1: $f(x+h) = (x + h) + 5 = \cancel{x} + h + \cancel{5}$

Step 2: $-f(x) = -[x+5] = \underline{\cancel{-x} \quad \cancel{-5}}$

h

Step 3: Add ① + ②

Step 4: Divide ③ by h: $h/h = 1$

Step 5: $\lim_{h \to 0}$ ④ $= \lim_{h \to 0} 1 = 1 = f'(x)$

Useful formulas: ① $(a \pm b)^2 = a^2 \pm 2ab + b^2$ ← Perfect Square

② $a^2 - b^2 = (a - b)(a + b)$ ← Difference of Square

Recall point-slope formula

Given m-slope and (x_1, y_1) - point

$y - y_1 = m(x - x_1)$

Given m=slope and (x_1, y_1) p...

$$y - y_1 = m(x - x_1)$$

In this class m is going to be the derivative.

$$y - y_1 = f'(x_1)(x - x_1)$$

Ex 2: Given $f(x) = x^2 - 3$.

ⓐ Find the slope of the tangent line. (i.e. Find $f'(x)$.)

Step 1: $f(x+h) = (x+h)^2 - 3 = \cancel{x^2} + 2xh + h^2 \cancel{-3}$

Step 2: $-f(x) \quad = -(x^2 - 3) \quad = \cancel{-x^2} \qquad \cancel{+3}$

Step 3: Add ① + ② $\qquad 2xh + h^2$

Step 4: Divide Step 3 by h.

$$\frac{2xh + h^2}{h} = \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x + h$$

Step 5: $\lim\limits_{h \to 0}$ ④ $= \lim\limits_{h \to 0} (2x + h) = 2x + 0 = 2x = f'(x)$

Game Plan: To find the equation of the tangent line to $f(x)$ at the point $x = c$, follow the following steps.

① Find $f'(x)$

② Calculate $f'(c)$

③ Calculate $f(c)$

④ Plug ② + ③ into the point-slope formula

$$y - f(c) = f'(c)(x - c)$$

ⓑ Find the equation of the tangent line to $f(x)$ at $x = 2$.

$$f(x) = x^2 - 3$$

Step 1: $f'(x) = 2x$ (from part a)

Step 2: $f'(2) = 2(2) = 4$

Step 3: $f(2) = 2^2 - 3 = 4 - 3 = 1$

Step 4: Point-Slope Formula

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$$y - f(2) = f'(2)(x-2)$$

$$y - 1 = 4(x-2)$$

$$y - 1 = 4x - 8$$

$$\underline{\;+1 \qquad\quad +1\;}$$

$$y = 4x - 7$$

Ex 2: Given $f(x)=x^2-3$.

ⓑ Find the eqn of the tangent line to $f(x)$ at $x=2$.

Step 1: $f'(x)=2x$ (from part a)

Step 2: $f'(2)=2(2)=4$

Step 3: $f(2)=(2)^2-3=4-3=1$

Step 4: Point-Slope Formula

$$y-f(2)=f'(2)(x-2)$$
$$y-1=4(x-2)$$
$$y-1=4x-8$$
$$+1 \quad\quad +1$$
$$y=4x-7$$