Lesson 9: Basic Differentiation Rules

Monday, September 15, 2025 10:25 AM

① **Constant Rule**: For any constant c,

$$\frac{d}{dx}[c] = 0$$

② **Power Rule**: For any real #, n,

$$\frac{d}{dx}(x^n) = n x^{n-1}$$

Ex 1: Find $f'(x)$ for $f(x) = x^2$

$$f'(x) = 2x^{2-1} = 2x$$

Ex 2: Find $f'(x)$ for $f(x) = x^{-4}$

$$f'(x) = -4x^{-4-1} = -4x^{-5} = \frac{-4}{x^5}$$

③ **Constant Multiple Rule**

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

④ + ⑤ **Sum/Difference Rule**

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

Ex 3: Find $f'(x)$ for $f(x) = x^5 + 5x^2$

$$f'(x) = \frac{d}{dx}[x^5 + 5x^2]$$

$$= \frac{d}{dx}[x^5] + 5\frac{d}{dx}[x^2]$$

$$= 5x^4 + 5(2x)$$

$$= 5x^4 + 5(2x)$$
$$= 5x^4 + 10x$$

$$\boxed{x^{-m} = \frac{1}{x^m}}$$

Ex 4: Find $f'(x)$ for $f(x) = \frac{3}{x^4} - 2x^2 + 6x + 7$
$$= 3x^{-4} - 2x^2 + 6x + 7$$

$$f'(x) = 3(-4)x^{-4-1} - 2(2)x^{2-1} + 6 + 0$$
$$= -12x^{-5} - 4x + 6$$
$$= -\frac{12}{x^5} - 4x + 6$$

Derivative of Natural Exponential: $\frac{d}{dx}(e^x) = e^x$

Product Rule: $h(x) = u(x)v(x)$
$$\frac{d}{dx}[h(x)] = \frac{d}{dx}[u(x)]v(x) + u(x)\frac{d}{dx}[v(x)]$$
$$= u'(x)v(x) + u(x)v'(x)$$

Ex 5: Given $h(x) = 2x^3e^x$. Compute $h'(x)$.

$u(x) = 2x^3$ $\quad$ $v(x) = e^x$

$u'(x) = 6x^2$ $\quad$ $v'(x) = e^x$

By product rule,
$$h'(x) = u'(x)v(x) + u(x)v'(x)$$
$$= 6x^2e^x + 2x^3e^x$$
$$= e^x(6x^2 + 2x^3)$$

Ex 6: Given $h(x) = \sqrt{x}(2x^2 + 4)$. Find $h'(x)$

Ex 6: Given $h(x)=\sqrt{x}\,(2x^2+4)$. Find $h'(x)$

Expand $h(x)$ to use Power Rule

$$h(x)=x^{1/2}(2x^2+4)$$

$$=2x^{1/2}x^2+4x^{1/2}$$

$$=2x^{5/2}+4x^{1/2}$$

$2/2=1$

$$h'(x)=2\left(\frac{5}{2}\right)x^{5/2-2/2}+4\left(\frac{1}{2}\right)x^{1/2-2/2}$$

$$=5x^{3/2}+2x^{-1/2}$$

Quotient Rule: $h(x)=\dfrac{u(x)}{v(x)}$

$$h'(x)=\frac{u'(x)v(x)-u(x)v'(x)}{v^2(x)}$$

Ex 7: Let $h(x)=\dfrac{1}{x^2}$. Find $h'(x)$

Method 1: Power Rule

$$h(x)=x^{-2}\Rightarrow h'(x)=-2x^{-2-1}=-\frac{2}{x^3}$$

Method 2: Quotient Rule

$u(x)=1$ $\qquad$ $v(x)=x^2$

$u'(x)=0$ $\qquad$ $v'(x)=2x$

$$h'(x)=\frac{u'(x)v(x)-u(x)v'(x)}{v^2(x)}=\frac{0(x^2)-1(2x)}{(x^2)^2}$$

$$=\frac{0-2x}{x^4}=\frac{-2x}{x^4}=-\frac{2}{x^3}$$

$$= \frac{\ldots}{x^4} = \frac{\ldots}{x^4} = -\frac{\ldots}{x^3}$$

Ex 8: Let $h(x) = \dfrac{x^2+1}{x^3-3x}$. Find $h'(x)$

$u(x) = x^2+1 \qquad v(x) = x^3-3x$

$u'(x) = 2x \qquad v'(x) = 3x^2-3$

$$h'(x) = \frac{u'(x)\,v(x) - u(x)\,v'(x)}{v^2(x)}$$

$$= \frac{2x(x^3-3x) - \boxed{(x^2+1)(3x^2-3)}}{(x^3-3x)^2}$$

To do the purple square

	x^2	$+1$
$3x^2$	$3x^4$	$\cancel{3x^2}$
-3	$\cancel{-3x^2}$	-3

$$= \frac{2x^4-6x^2-(3x^4-3)}{(x^3-3x)^2}$$

$$= \frac{2x^4-6x^2-3x^4+3}{(x^3-3x)^2}$$

$$= \frac{-x^4-6x^2+3}{(x^3-3x)^2}$$