Lesson 11: The Chain Rule Part 1

Friday, September 19, 2025 10:24 AM

Problems to know

(Some solutions below)

pg. 2 Problem (iv) $e^{2x^2-7x-15}=1$ Answer: $x=5,-3/2$

Problem (b)(i) $\dfrac{\log_{15}(9)+\log_{15}(25)}{e^{3\ln(5)+2\ln(7)}}$ Answer: $\dfrac{2}{5^3\cdot 7^2}$

pg 4 Problem (v) $\sin^{-1}\left(\cos\left(\frac{3\pi}{5}\right)\right)$ Answer: $-\pi/10$

Problem (vi) $\sin^{-1}\left(\cos\left(\frac{4\pi}{7}\right)\right)$ Answer: $-\pi/14$

pg 8 Problem (ii) $\sin(x)=\cos(2x)$ Answer: $\pi/6,\ \frac{5\pi}{6},\ \frac{3\pi}{2}$

Problem (iii) $-\sin^2(x)=\cos(2x)$ Answer: $\frac{\pi}{2},\ \frac{3\pi}{2}$

Recall composition of functions.

Let $y=f(g(x)) \Rightarrow$ g - inner function
f - outer function

Ex 1: Determine f and g for $y=(3x+1)^2$

$f(x)=x^2$

$g(x)=3x+1$

Check $y=f(g(x))=f(3x+1)=(3x+1)^2$ ✓

EX 2: Determine f and g for $y=\sin^2 x=(\sin x)^2$

$f(x)=x^2$

$g(x)=\sin x$

Check $y=f(g(x))$

Ex 3: Determine f and g for $y=\ln(2x)$

$f(x)=\ln(x)$

$f(x) = \ln(x)$
$g(x) = 2x$

Check $y = f(g(x))$

Ex 4: ~~Determine~~ Determine f and g for $y = \sqrt[3]{2x+1}$

$f(x) = \sqrt[3]{x}$
$g(x) = 2x+1$

Check $y = f(g(x))$

Chain Rule

Let $y = f(g(x))$

version 1: $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$ ☆

Version 2: $\frac{dy}{dx} = \frac{d}{dx}[f(\underbrace{g(x)}_{=u})] = \frac{df}{du} \cdot \frac{du}{dx}$

Ex 1: Find y' of $y = (3x+1)^2$

Expand $y = (3x)^2 + 2(3x)(1) + 1^2$
$= 9x^2 + 6x + 1$
$y' = 9(2)x^1 + 6 + 0 = 18x + 6$

Ex 2: Find y' of $y = 2\cos^3 x$

Rewrite $y = 2(\cos(x))^3$

$f = 2x^3 \quad g = \cos(x)$
$f' = 6x^2 \quad g' = -\sin(x)$

By Chain Rule, $y' = f'(g(x)) \cdot g'(x)$

By Chain Rule,

$$y' = f'(g(x)) \cdot g'(x)$$
$$= f'(\cos(x)) \cdot (-\sin(x))$$
$$= 6(\cos(x))^2(-\sin(x))$$
$$= -6\cos^2(x)\sin(x)$$

Ex 3: Find y' of $y = \left(\frac{2x}{3x^2+x}\right)^3$

Simplify and rewrite y.

$$y = \left(\frac{2\cancel{x}}{\cancel{x}(3x+1)}\right)^3$$
$$= \left(\frac{2}{3x+1}\right)^3$$
$$= \frac{2^3}{(3x+1)^3}$$
$$= \frac{8}{(3x+1)^3}$$
$$= 8(3x+1)^{-3}$$

Find y'.

$f(x) = 8x^{-3}$ $\quad g(x) = 3x+1$

$f' = 8(-3)x^{-4}$ $\quad g' = 3$

$= -24x^{-4}$

By Chain Rule

$$y' = f'(g(x)) \cdot g'(x)$$
$$= f'(3x+1) \cdot 3$$
$$= -24(3x+1)^{-4} \cdot 3$$
$$= \frac{-72}{(3x+1)^4}$$

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pg. 2 Problem (iv) $e^{2x^2-7x-15} = 1$

Problem (b)(i) $\dfrac{\log_{15}(9) + \log_{15}(25)}{e^{3\ln(5)+2\ln(7)}} = \dfrac{\cancel{\log_{15}}(\overbrace{9 \cdot 25}^{\overbrace{3^2 \cdot 5^2}^{(3 \cdot 5)^2}})}{\underbrace{e^{\ln(5^3)+\ln(7^2)}}_{\cancel{e}^{\cancel{\ln}(5^3 \cdot 7^2)}}} = \dfrac{2}{5^3 \cdot 7^2}$

pg 4 Problem (v) $\sin^{-1}\left(\cos\left(\frac{3\pi}{5}\right)\right)$

Problem (vi) $\sin^{-1}\left(\cos\left(\frac{4\pi}{7}\right)\right)$

$\cos(x) = \sin\left(\frac{\pi}{2} - x\right)$

(v) $\sin^{-1}\left(\underbrace{\cos\left(\frac{3\pi}{5}\right)}\right)$

$$(v)\ \sin^{-1}\Big(\underbrace{\cos\left(\tfrac{3\pi}{5}\right)}_{\textstyle \sin\left(\tfrac{\pi}{2}-\tfrac{3\pi}{5}\right) = \sin\left(\tfrac{5\pi}{10}-\tfrac{6\pi}{10}\right) = \sin\left(-\tfrac{\pi}{10}\right)}\Big)$$

$$= \cancel{\sin^{-1}}\left(\cancel{\sin}\left(-\tfrac{\pi}{10}\right)\right) = -\pi/10$$

pg 8 Problem (ii) $\sin(x) = \cos(2x)$

Problem (iii) $-\sin^2(x) = \cos(2x)$

$\cos(2x) = 1 - 2\sin^2(x)$

This problems are $[0, 2\pi)$

(ii) $\sin(x) = 1 - 2\sin^2(x)$

$2\sin^2(x) + \sin(x) - 1 = 0$

$u = \sin(x)$

-2

$\wedge$

$2\ -1 = 1$

$2u^2 + 1u - 1 = 0$

$2u^2 + 2u - u - 1 = 0$

$2u(u+1) - (u+1) = 0$

$(2u-1)(u+1) = 0$

$u = \frac{1}{2} \qquad u = -1$

$\sin\theta = \frac{1}{2} \qquad \sin\theta = -1$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6} \qquad \theta = \frac{3\pi}{2}$

(iii) $-\sin^2(x) = 1 - 2\sin^2(x)$

$\sin^2(x) = 1$

$\sin(x) = \pm 1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$