Review Session 1

Monday, September 22, 2025   10:26 AM

Problems to know

Set 4: pg. 9 (i) $\cos\left(\sin^{-1}\left(\frac{x}{3}\right)\right)$

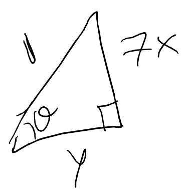
(iii) $\cos(\sin^{-1}(7x))$

Set 5: pg. 12 (b) $f(x) = \frac{x+5}{x-2}$ over $(2, \infty)$

pg. 13 (d) $f(x) = \frac{2x+1}{x+1}$ over $(-1, 0)$

pg. 9 (iii) $\cos(\underbrace{\sin^{-1}(7x)}_{\theta}) = \cos\theta$

$\sin\theta = \frac{7x}{1} = \frac{opp}{hyp}$



Find y.

$(7x)^2 + y^2 = 1^2$

$49x^2 + y^2 = 1$

$y^2 = 1 - 49x^2$

$y = \sqrt{1-49x^2}$

$\cos\theta = \frac{adj}{hyp} = \sqrt{1-49x^2}$

pg. 12 $f(x) = \frac{2x+1}{x+1}$ over the domain $(-1, 0)$

Abusing domain of f to get range of f $\shortparallel$ domain of f^{-1}

(a) inverse function

$y = \frac{2x+1}{x+1}$

Solve for x.

$y(x+1) = 2x+1$

$yx + y = 2x + 1$

$yx - 2x = -y + 1$

$x(y-2) = -y + 1$

$x = \frac{-y+1}{y-2} = \boxed{\frac{y-1}{2-y} = x}$

$f^{-1}(x) = \frac{x-1}{2-x}$

(b) domain of inverse

Since f has domain $(-1, 0)$

We need $x < 0$ and $x > -1$

$\frac{y-1}{2-y} < 0$

$y - 1 < 0$ and $\boxed{2 - y > 0}$

$y < 1$ and $2 > y$, $y < 2$

Combine to where both happen

$y < 1$

$\frac{y-1}{2-y} > -1$

$y - 1 > -1(2-y)$

$y - 1 > -2 + y$

$-1 > -2$

True. But its true everywhere

$y < 1$

Domain of f^{-1}: $y < 1 \Rightarrow (-\infty, 1)$

$f^{-1}(x) = \frac{x-1}{2-x}$ Domain of f^{-1}: $y<1 \Rightarrow (-\infty, 1)$

~~Set 6: pg. 15~~ (vii) $\lim_{x\to-\infty} \left(\sqrt{x^2+3x} + x\right)$

pg. 16 (ix) $\lim_{x\to 0} \frac{\sin(3x)}{7x}$ (Know how to do w/ different #)

(x) $\lim_{x\to\infty} \frac{\sin(x)}{\ln(x)}$

pg. 17 (xi) $\lim_{x\to\infty} \frac{\cos(3x)}{\ln(x)}$

pg. 18 (xvi) $\lim_{x\to-\infty} \frac{x^3+3x^2}{2x^3+\sqrt{9x^6+4x^4}}$

pg. 19 (xvii) $\lim_{x\to 0^-} \frac{x^3+3x^2}{2x^3+\sqrt{9x^6+4x^4}}$

(vii) $\lim_{x\to-\infty} \frac{\left(\sqrt{x^2+3x} + x\right)}{1} \cdot \frac{\left(\sqrt{x^2+3x} - x\right)}{\left(\sqrt{x^2+3x} - x\right)}$ $a^2-b^2=(a-b)(a+b)$

$= \lim_{x\to-\infty} \frac{x^2+3x-x^2}{\sqrt{x^2+3x}-x}$

$= \lim_{x\to-\infty} \frac{3x}{\sqrt{x^2+3x}-x}$

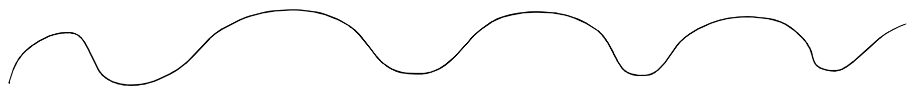
$\approx \lim_{x\to-\infty} \frac{3x}{\sqrt{x^2}-x}$ $\sqrt{x^2}$ and $x\to-\infty$ that means $\sqrt{x^2} = -x$

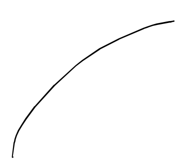
$= \lim_{x\to-\infty} \frac{3x}{-x-x}$

$= \lim_{x\to-\infty} \frac{3x}{-2x}$

$= \lim_{x\to-\infty} \frac{3}{-2} = -\frac{3}{2}$

(x) $\lim_{x\to\infty} \frac{\sin(x)}{\ln(x)} = \frac{\#}{\infty} = 0$

Sin(x) Graph 

ln(x) Graph 

$x\to\infty$
$y\to\infty$

$$(xvi)\ \lim_{x\to-\infty} \frac{x^3+3x^2}{2x^3+\sqrt{9x^6+4x^4}}$$

$$\approx \lim_{x\to-\infty} \frac{x^3}{2x^3+\sqrt{9x^6}}$$

$\sqrt{x^6}$ and $x\to-\infty$

$\sqrt{x^6} = -x^3$

$$= \lim_{x\to-\infty} \frac{x^3}{2x^3-3x^3}$$

$$= \lim_{x\to-\infty} \frac{\cancel{x^3}}{-\cancel{x^3}} = -1$$

$$(xvii)\ \lim_{x\to 0^-} \frac{x^3+3x^2}{2x^3+\sqrt{9x^6+4x^4}}$$

Pin

Will do on Wednesday