Wednesday, September 24, 2025 10:25 AM

pg. 19 (xvii) $\lim\limits_{x\to 0^-} \dfrac{x^3+3x^2}{2x^3+\sqrt{9x^6+4x^4}}$

**Set 7:** (a) [pg. 20] Using IVT determine which intervals $x^3-3x=5$ has a root?

A) $(0,1)$, B) $(1,2)$ C) $(2,3)$ D) $(3,4)$ E) $(4,5)$

pg. 21 (c) Same question but $x^3-3x^2=-2x+1$

A) $(-1,0)$, B) $(0,1)$, C) $(1,2)$, D) $(2,3)$, E) $(3,4)$

**Set 8:** pg. 23 (b) $f'(5)=\lim\limits_{h\to 0}\dfrac{f(5+h)-f(5)}{h}=3$

Find $\lim\limits_{h\to 0}\dfrac{f(5+2h)-f(5-4h)}{h}$

pg. 24. (e) $\lim\limits_{h\to 0}\dfrac{\ln(1+3h)}{7h}$

**Set 9:** pg. 25 (a) and (b)

**Set 10:** pg. 28 (vi)

**Set 11:** pg. 30 (c)

**Set 12:** (nada)

**Set 13:** pg. 35 (i) and (ii)

(xvii) $\lim\limits_{x\to 0^-} \dfrac{x^3+3x^2}{2x^3+\sqrt{9x^6+4x^4}}$

$= \lim\limits_{x\to 0^-} \dfrac{x^3+3x^2}{2x^3+\sqrt{x^4(9x^2+4)}}$

Again b/c $x\to 0^- \Rightarrow \boxed{\sqrt{x^4}=-x^2}$

$= \lim\limits_{x\to 0^-} \dfrac{x^3+3x^2}{2x^3-x^2\sqrt{9x^2+4}}$

$= \lim\limits_{x\to 0^-} \dfrac{\cancel{x^2}(x+3)}{x^2(2x-\sqrt{9x^2+4})}$

$$= \lim_{x\to 0^-} \frac{\quad}{x^2(2x - \sqrt{9x^2+4})}$$

$$= \lim_{x\to 0^-} \frac{(x+3)}{(2x - \sqrt{9x^2+4})}$$

$$= \lim_{x\to 0^-} \frac{0+3}{2(0) - \sqrt{0+4}} = \frac{3}{-2} = -\frac{3}{2}$$

Set 7: (a) Use the IVT for $x^3 - 3x = 5$ to determine which interval has a root

A) $(0,1)$

B) $(1,2)$

C) $(2,3)$

D) $(3,4)$

E) $(4,5)$

We want $f(a) = +$ and $f(b) = -$ for $(a,b)$ or vice versa

Find $f(x)$. [need $f(x) = 0$

$$x^3 - 3x = 5$$

$$\underbrace{x^3 - 3x - 5}_{f(x)} = 0$$

$$f(0) = 0^3 - 3(0) - 5 = -$$

$$f(1) = 1^3 - 3(1) - 5 = -$$

$$f(2) = 2^3 - 3(2) - 5 = -$$

$$f(3) = 3^3 - 3(3) - 5 = +$$

Set 8: $f'(5) = \lim_{h\to 0} \frac{f(5+h) - f(5)}{h} = 3$

Then find $\lim_{h\to 0} \frac{f(5+2h) - f(5-4h)}{h}$

$$= \lim_{h\to 0} \frac{f(5+2h) - f(5) + f(5) - f(5-4h)}{h}$$

$$= \lim_{h\to 0} \frac{2(f(5+2h) - f(5))}{2(h)} + \lim_{h\to 0} \frac{f(5) - f(5-4h)}{h}$$

$$= 2 \lim_{h\to 0} \frac{f(5+2h) - f(5)}{2h} - \lim_{h\to 0} \frac{(-4)(f(5-4h) - f(5))}{(-4)h}$$

If $h \to 0$, then $2h \to 0$, $H \to 0$

$$= 2 \lim_{H\to 0} \frac{f(5+H) - f(5)}{H} - (-4) \lim_{h\to 0} \frac{f(5-4h) - f(5)}{(-4)h}$$

$$= 2 f'(5) \qquad + 4 \lim_{J\to 0} \frac{f(5+J) - f(5)}{J}$$

$$= 2f'(5) + 4f'(5)$$
$$= 6f'(5) = 6(3) = \boxed{18}$$

(e) $$\lim_{h \to 0} \frac{\ln(\overset{x}{1} + \overset{H}{\overbrace{3h}})}{7h} \qquad f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(\ln(1+3h) - \ln(1))}{3(7h)}$$

$$= \frac{3}{7} \cdot \lim_{h \to 0} \frac{\ln(1+3h) - \ln(1)}{3h} \qquad H = 3h$$

$$= \frac{3}{7} \cdot \lim_{H \to 0} \frac{\ln(1+H) - \ln(1)}{H} = f'(1) \text{ where } \begin{array}{l} f(x) = \ln(x) \\ f'(x) = \frac{1}{x} \end{array}$$

$$= \frac{3}{7} \cdot \left(\frac{1}{1}\right) = \frac{3}{7}$$

Set 11: pg. 30 (c)

Eqn of tangent line of $f(x) = \tan(x)$ over $(-\pi/2, 0)$ which is parallel to line $y = 2x+1$

$$\begin{array}{c|c} S & A \\ \hline T & C \end{array}$$

$m = f'(x) = \sec^2(x)$

Parallel to $y = 2x+1$

$f'(x) = 2 = \sec^2(x)$

$\pm\sqrt{2} = \sec(x)$

$\pm \frac{1}{\sqrt{2}} = \cos(x)$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

BUT $(-\pi/2, 0)$

$x = -\frac{\pi}{4}$

Find $y$ when $x = -\frac{\pi}{4}$

$y = f(x) = f\left(-\frac{\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right)$

$= -\tan\left(\frac{\pi}{4}\right)$

$= -1 \quad \Rightarrow \left(-\frac{\pi}{4}, -1\right)$

$y - y_1 = f'(x_1)(x - x_1)$

$y - (-1) = 2\left(x - \left(-\frac{\pi}{4}\right)\right)$

$y + 1 = 2\left(x + \frac{\pi}{4}\right)$

$y = 2x + \frac{2\pi}{4} - 1$

$y = 2x + \frac{\pi}{2} - 1$

$$y = 2x + \frac{\pi}{2} - 1$$