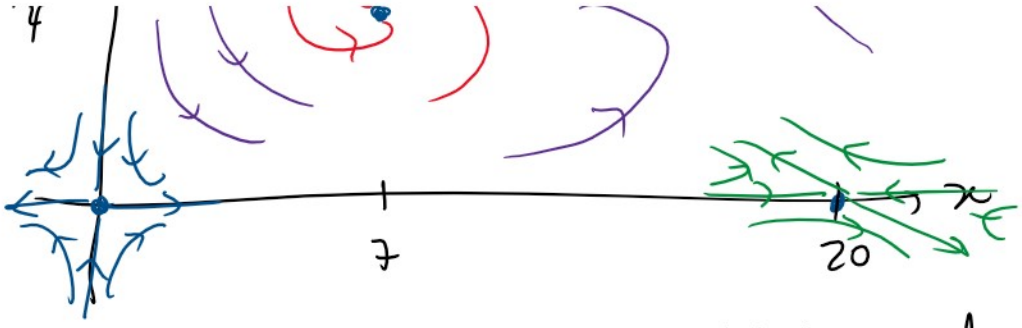
$\textcircled{3}\ \left(7, \frac{5}{13}\right)$ $\quad \underline{\underline{J}} = \begin{bmatrix} \text{[illegible]} & \\ -7/5 & -14 \end{bmatrix}$ $\quad y = -\frac{5}{7} \mp 1.1$

$\textcircled{3}\ (50, 0)$ $\quad \underline{\underline{J}} = \begin{bmatrix} 0 & 3d \\ -10 & -70 \end{bmatrix}$ $\quad y = 3d^2 - 10$

$\begin{matrix} 0 & -51 \\ & 0 \end{matrix}\Big]$ $\quad y = -51/10$



5. Interpret in terms of rabbits and foxes

as $t \to \infty$

$x \longrightarrow 7$ (rabbits)

$y \longrightarrow \frac{13}{4}$ (foxes)

populations converge to a single value

"coexistence"

II. Competitors:

$x(t)$ - rabbits $\quad$ $y(t)$ - deer

- both eat vegetation
- neither preys on the other

> competition model

Equations:

$$\frac{dx}{dt} = a_1 x - b_1 x^2 - c_1 xy$$

$$\frac{dy}{dt} = a_2 y - b_2 y^2 - c_2 xy$$

$a_1 x - b_1 x^2$: logistic growth

$-c_1 xy$, $-c_2 xy$: competition for resources (both negative)

Note: $a_i, b_i, c_i > 0$

Rewrite:

$$x' = x(a_1 - b_1 x - c_1 y)$$

$$y' = y(a_2 - b_2 y - c_2 x)$$

This system has four critical points:

$\rightarrow (0,0)$

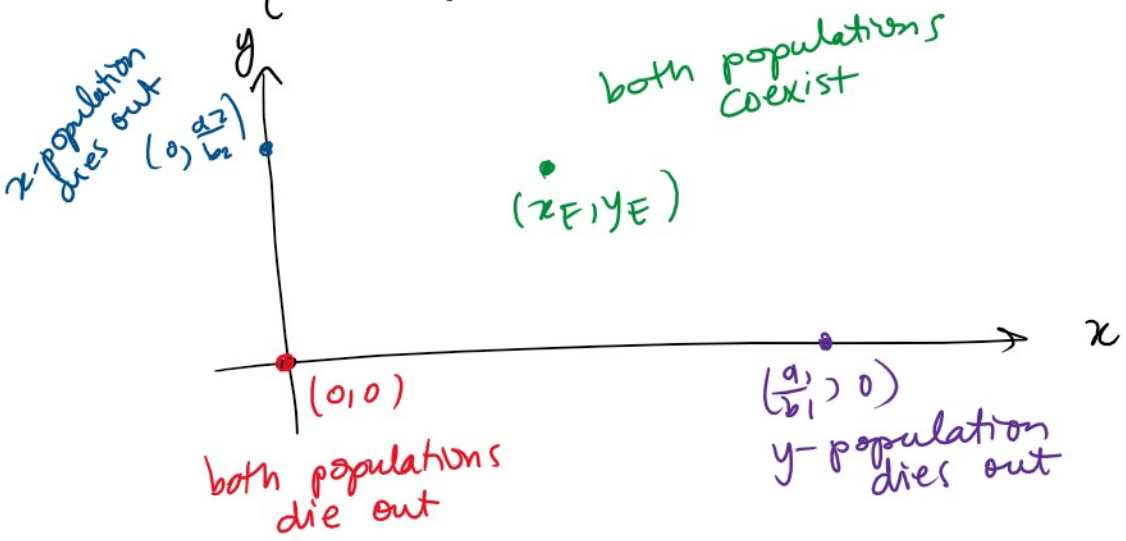
This system has four critical P

$x=0$, and $y=0 \longrightarrow (0,0)$

if $x=0 \longrightarrow a_2 - b_2 y = 0 \longrightarrow \left(0, \frac{a_2}{b_2}\right)$

if $y=0 \longrightarrow a_1 - b_1 x = 0 \longrightarrow \left(\frac{a_1}{b_1}, 0\right)$

if $\begin{cases} a_1 - b_1 x - c_1 y = 0 \\ a_2 - b_2 y - c_2 x = 0 \end{cases}$ call the solution (x_E, y_E)



Often coexistence is a goal.

WANT: (x_E, y_E) to be stable

Coexistence Criteria:

If $c_1 c_2 < b_1 b_2$ then (x_E, y_E) is an asymptotically stable critical point

$c_1 c_2$: Competition; $b_1 b_2$: inhibition

then the 2 species coexist.

Ex: $x' = 30x - 3x^2 + xy = x(30 - 3x + y)$ ← to calculate J

$y' = 60y - 3y^2 + 4xy = y(60 - 3y + 4x)$

Find the critical point for coexistence: (x_E, y_E)

$30 - 3x + y = 0$

$y = 3x - 30$

$60 - 3y + 4x = 0$

$60 - 3(3x - 30) + 4x = 0$

$60 - 9x + 90 + 4x = 0$

$150 = 5x$

$x = 30$

$y = 3(30) - 30$

$y = 60$

$\boxed{(x_E, y_E) = (30, 60)}$

WANT: (30,60) to be stable or asymptotically stable

@ (30,60) $\quad \underline{J} = \begin{bmatrix} -90 & 30 \\ 240 & -180 \end{bmatrix}$ $\quad \lambda = -15\,(9 \pm \sqrt{41})$

$\lambda < 0$

improper nodal sink

asymptotically stable

→ coexistence