Lecture 1

☆ Overview & Section 5.1

MA 303

Warm up: Which of the following equations are ODEs?
(check all that apply)

(a) $\frac{dy}{dt} = 3y$ ✓

(b) $y^2 = e^{7t} + 10t$ X

(c) $\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2}$ X $y(t,x)$ PDE

(d) $y'' + 3y' + 7y = \cos(t)$ ✓

MA 303: Differential Eqns & PDEs

ODEs: (Ordinary Diff. Eqn) relates a function of one variable with its derivatives

Ex: $\frac{dy}{dt} = 3y$ $\qquad y'' + 3y' + 7y = \cos(t)$

PDEs: (Partial Differential Eqn) relates a function of multiple variables with its partial derivatives

Ex: $\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2}$ $\qquad$ Here $y(t,x)$

Today: Chapter 5

GOAL: Solve linear systems of ODEs

Recall: We can rewrite a higher order ODE as a 1st order system

Recall: We can [illegible]
a 1st order system

Ex: $y'' + 3y' + 7y = 0$

System: Let $x_1 = y$ $\quad x_2 = y'$

$$x_1' = y' = x_2$$

$$x_2' = y'' = -3y' - 7y = -3x_2 - 7x_1$$

System: $\begin{cases} x_1' = x_2 \\ x_2' = -3x_2 - 7x_1 \end{cases}$

Matrix Form

$$\underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}'}_{\underline{x}\ \text{Vector}} = \underbrace{\begin{bmatrix} 0 & 1 \\ -7 & -3 \end{bmatrix}}_{\underline{\underline{A}}\ \text{coefficient matrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\underline{x}' = \underline{\underline{A}}\,\underline{x}$$

Section 5.1 – review of linear algebra + systems

I. Matrices:

A $m \times n$ matrix $\underline{\underline{A}}$

$$\underline{\underline{A}} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$a_{\text{row, col}}$

m rows

n columns

Notation: $\underline{\underline{A}}$ is a matrix $m \times n$
uppercase letters

$\underline{x}$ is a vector $(n \times 1)$ $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$
lowercase letters

(a) matrix addition

$$\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} + \begin{bmatrix}1 & 2\\ 1 & 2\end{bmatrix} = \begin{bmatrix}1+1 & 2+2\\ 3+1 & 4+2\end{bmatrix} = \begin{bmatrix}2 & 4\\ 4 & 6\end{bmatrix}$$

(b) scalar multiplication

$$2\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} = \begin{bmatrix}2\cdot 1 & 2\cdot 2\\ 2\cdot 3 & 2\cdot 4\end{bmatrix} = \begin{bmatrix}2 & 4\\ 6 & 8\end{bmatrix}$$

scalar

multiply each element by 2

(c) Transpose of a matrix $\underline{A}^T$

row $\longrightarrow$ column

$$\underset{(2\times 2)}{\underline{A}} = \begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} \qquad \underset{(2\times 2)}{\underline{A}^T} = \begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}$$

$$\underset{(2\times 3)}{\underline{B}} = \begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix} \qquad \underline{B}^T = \begin{bmatrix}1 & 4\\ 2 & 5\\ 3 & 6\end{bmatrix} \ (3\times 2)$$

II Matrix Multiplication

$\underline{A}$ $(m\times p)$ $\qquad$ $\underline{B}$ $(p\times n)$

Then $\underline{A}\,\underline{B}$ is defined $\longrightarrow$ $(m\times n)$

$$\underline{A} = \begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix} \ (2\times 3) \qquad \underline{B} = \begin{bmatrix}1 & 1\\ 2 & 1\\ 1 & 1\end{bmatrix} \ (3\times 2)$$

$$\underset{(2\times 3)(3\times 2)}{\underline{A}\,\underline{B}} = \begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 1\\ 2 & 1\\ 1 & 1\end{bmatrix} = \underset{(2\times 2)}{\begin{bmatrix} & \\ & \end{bmatrix}}$$

$$= \begin{bmatrix}1\cdot 1 + 2\cdot 2 + 3\cdot 1 & 1\cdot 1 + 2\cdot 1 + 3\cdot 1\\ 4\cdot 1 + 5\cdot 2 + 6\cdot 1 & 4\cdot 1 + 5\cdot 1 + 6\cdot 1\end{bmatrix} = \begin{bmatrix}8 & 6\\ 20 & 15\end{bmatrix} = \underset{(2\times 2)}{\underline{A}\,\underline{B}}$$

Ex: $\underline{\underline{A}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ $\quad$ $\underline{\underline{C}} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\underset{(2\times 3)\,(2\times 2)}{\underline{\underline{A}}\,\underline{\underline{C}}} = \times$ NOT Defined

$\times$

$\checkmark$

$\underset{(2\times 2)\,(2\times 3)}{\underline{\underline{C}}\,\underline{\underline{A}}} = \begin{bmatrix} & & \\ & & \end{bmatrix} \; (2\times 3)$ this is defined

POLL: $\underline{\underline{A}} = \overset{(3\times 3)}{\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}}$ $\quad$ $\underline{\underline{B}} = \overset{(2\times 3)}{\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}}$

Which of the following matrix mult. is valid?

$\times$ (a) $\underline{\underline{A}}\,\underline{\underline{B}}$ $(3\times 3)(2\times 3)$ $\quad$ (c) neither

$\checkmark$ (b) $\underline{\underline{B}}\,\underline{\underline{A}}$ $(2\times 3)(3\times 3)$ $\quad$ (d) both $\times$

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III. Determinant:

If $\underline{\underline{A}}$ is a 2×2 matrix then its determinant

$$\det(\underline{\underline{A}}) = |\underline{\underline{A}}| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

Ex: $\det\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} = 1\cdot 3 - 2\cdot 4 = 3 - 8 = -5$

For determinant of bigger square matrices, we use Expansion by Minors

If $\underline{\underline{A}}$ is an $n\times n$ matrix

... $(n-1)\times(n-1)$ matrix obtained by