Riemann sums: $f(x) = x^2 + 2$

$n = 10$

$[0, 10] = [a, b]$

Left Sum: $\sum_{i=0}^{n-1} f(x_i)\Delta x$

$\Delta x = \frac{b-a}{n}$

Right Sum: $\sum_{i=1}^{n} f(x_i)\Delta x$

$x_i = a + i\Delta x$

$f(x) = x^2 + 2$

$n = 10$

$[0, 10] = [a, b]$

$\Delta x = \frac{10 - 0}{10} = 1$

$x_i = 0 + i \cdot 1 = i$

$f(x_i) = i^2 + 2$

<u>Left</u>: $\sum_{i=0}^{9} (i^2 + 2) \cdot 1$

<u>Right</u>: $\sum_{i=1}^{10} (i^2 + 2) \cdot 1$

Say they want Right - Left estimate.

$$L_{10} = \sum_{i=0}^{9} a_i \qquad R_{10} = \sum_{i=1}^{10} a_i$$

$$R_{10} = \cancel{a_1} + \cancel{a_2} + \cancel{a_3} + \dots + \cancel{a_8} + \cancel{a_9} + a_{10}$$

$$-\left(L_{10} = a_0 + \cancel{a_1} + \cancel{a_2} + \dots + \cancel{a_8} + \cancel{a_9}\right)$$

$$R_{10} - L_{10} = a_{10} - a_0$$

Looking back to our problem.

$$R_{10} - L_{10} = \sum_{i=1}^{10}(i^2+2) - \sum_{i=0}^{9}(i^2+2)$$

$$= 10^2 + 2 - (0^2+2) = 10^2 = 100$$

Trapezoid Rule: $f(x) = 2x+3$

$n = 4$

$[1, 9]$

① $\Delta x = \frac{9-1}{4} = \frac{8}{4} = 2$

② $x_0 = 1$ $\quad f(x_0) = 5$

$x_1 = 3$ $\quad f(x_1) = 6+3 = 9$

$x_2 = 5$ $\quad f(x_2) = 10+3 = 13$

$x_3 = 7$ $\quad f(x_3) = 14+3 = 17$

$x_4 = 9$ $\quad f(x_4) = 18+3 = 21$

$$\boxed{\begin{aligned} f(x_0) &= 5 \\ 2f(x_1) &= 18 \\ 2f(x_2) &= 26 \\ 2f(x_3) &= 34 \\ f(x_4) &= 21 \end{aligned}}$$

Sum = ?

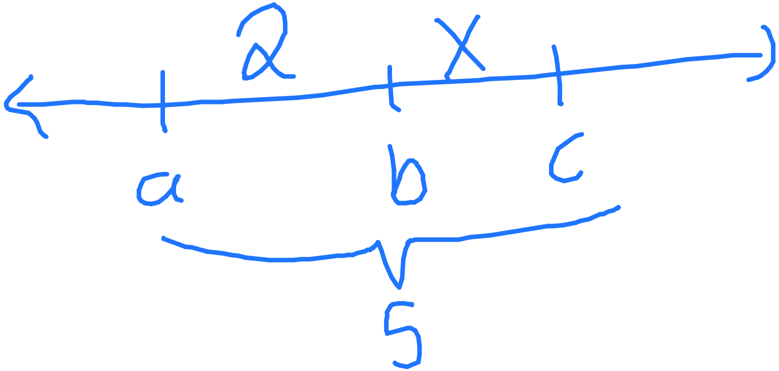
Trapezoid Rule

$$T_4 = \frac{1}{2}\Delta x \cdot \text{Sum} = \frac{1}{2} \cdot 2 \cdot \text{Sum} = \text{Sum}$$

$$= 5+18+26+34+21$$

Definite Integrals:

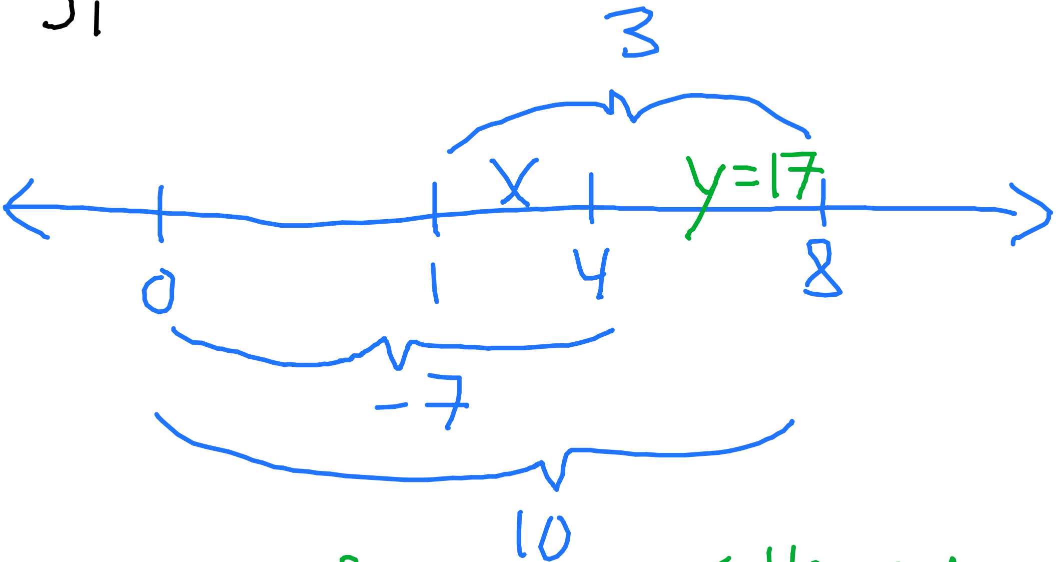
Say $\int_a^c f(x)\,dx = 5$ and $\int_a^b f(x)\,dx = 2$. Find $\int_b^c f(x)\,dx$.



$2 + x = 5 \rightarrow x = 3$

$\int_1^8 f(x)\,dx = 3, \int_0^4 f(x)\,dx = -7, \int_0^8 f(x)\,dx = 10,$

Find $\int_1^4 f(x)\,dx.$



$$\int_4^8 f(x)\,dx = \int_0^8 f(x)\,dx - \int_0^4 f(x)\,dx = 10-(-7)$$
$$= 17$$

$$\int_1^4 f(x)\,dx = \int_1^8 f(x)\,dx - \int_4^8 f(x)\,dx = 3 - 17$$
$$= \boxed{-14}$$

Exponential Growth/Decay

$$\frac{dy}{dt} = yt \Rightarrow y = Ce^{kt}$$

answer choice

good for calculator

$$\text{half-life} \Rightarrow k = \frac{\ln(1/2)}{\text{half-life}} = \frac{-\ln(2)}{\text{half-life}}$$

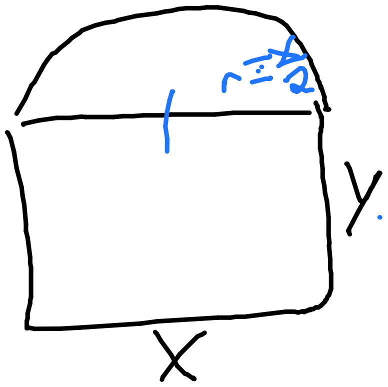
Try #19 and #20 from Practice Exam
After 3

We also cover Exam 1 #4

Exam 3 #5

Exam 3 Practice # 8 | Norm Window. $P = 10$

Find x.



$$A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$$
$$= xy + \frac{1}{2}\pi\frac{x^2}{4}$$
$$= xy + \frac{1}{8}\pi x^2$$

$$10 = P = x + 2y + \pi\left(\frac{x}{2}\right)$$
$$20 = 2x + 4y + \pi x$$
$$\frac{20 - 2x - \pi x}{4} = \frac{4y}{4}$$
$$y = \frac{20 - 2x - \pi x}{4}$$

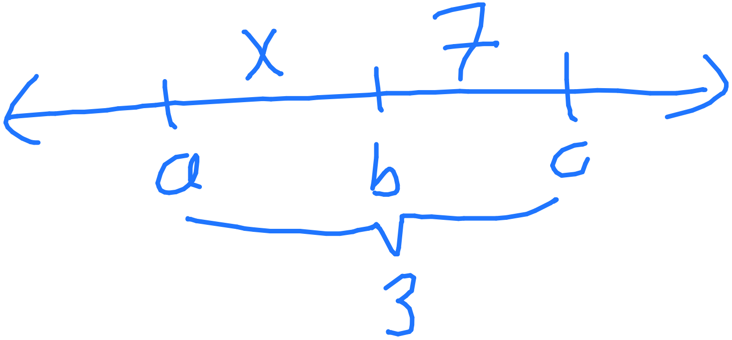
Plug y into A.

$$A = x\left(\frac{20 - 2x - \pi x}{4}\right) + \frac{1}{8}\pi x^2$$
$$= \frac{1}{4}\left(20x - 2x^2 - \pi x^2\right) + \frac{1}{8}\pi x^2$$
$$A' = \frac{1}{4}\left(20 - 4x - 2\pi x\right) + \frac{2}{8}\pi x = 0$$
$$= 5 - x - \frac{1}{2}\pi x + \frac{1}{4}\pi x = 0$$
$$5 - x - \frac{1}{4}\pi x = 0$$
$$5 = x + \frac{1}{4}\pi x$$
$$5 = x\left(1 + \frac{1}{4}\pi\right)$$
$$5 = x\left(\frac{4 + \pi}{4}\right)$$
$$x = \frac{20}{4 + \pi}$$

A

Definite Integrals

Say $\int_a^c f(x)\,dx = 3$, $\int_b^c f(x)\,dx = 7$. Find $\int_a^b f(x)\,dx$.



$x + 7 = 3$

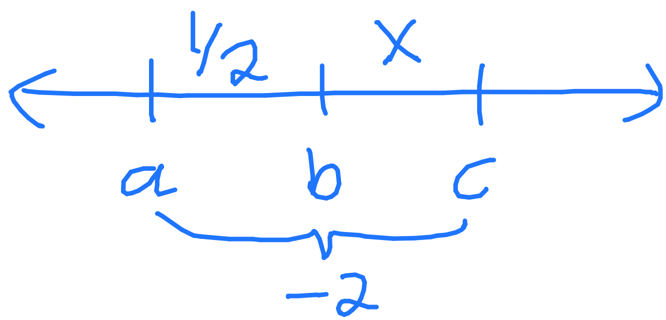
$x = -4 \Rightarrow \int_a^b f(x) = -4$

After Exam 3 Practice #5

$\int_a^b 2f(x)\,dx = 1$, $\int_a^c -\frac{1}{2}f(x)\,dx = 1$. Find $\int_b^c f(x)\,dx$.

$$\int_a^b 2f(x)\,dx = 1$$
$$2\int_a^b f(x)\,dx = 1$$
$$\int_a^b f(x)\,dx = \frac{1}{2}$$

$$\int_a^c -\frac{1}{2}f(x)\,dx = 1$$
$$-\frac{1}{2}\int_a^c f(x)\,dx = 1$$
$$\int_a^c f(x)\,dx = -2$$



$$\frac{1}{2} + x = -2 \rightarrow x = -2 - \frac{1}{2} = -\frac{5}{2}$$

HW 33 #2.2) $v(t) = 6t - 3$. Find time when the displacement is 0.

i.e. Solve $\int_0^t (6t-3)\,dt = 0$ for t.

$$\left(\frac{6t^2}{2} - 3t\right)\Big]_0^t = 0$$

$$(3t^2 - 3t)\Big]_0^t = 0$$

$$3t^2 - 3t - (\cancel{3\cdot 0 - 3\cdot 0}) = 0$$

$$3t^2 - 3t = 0$$

$$3t(t-1) = 0$$

$$t = 0, \quad t = 1$$

after it starts moving

Exponential Growth/Decay

$$\frac{dy}{dt} = ky \Rightarrow y = Ce^{kt}$$

$$\text{half-life} \Rightarrow k = \frac{\ln(1/2)}{\text{half-life}} = \frac{-\ln(2)}{\text{half-life}}$$

multiple choice

easier for calculator

HW 35. #4) $\frac{dP}{dt} = kP \Rightarrow P = Ce^{kt}$

$P = 30000$ $t = 3$ $\Rightarrow 30000 = Ce^{3k}$ ①

$P = 40000$ $t = 5$ $\Rightarrow 40000 = Ce^{5k}$ ②

$P(10) = ?$

Solve ② for C.

$$40000e^{-5k} = Ce^{5k}e^{-5k}$$

$$40000e^{-5k} = C$$

Plug C into ①.

$$3\cancel{0000} = 4\cancel{0000}e^{-5k}e^{3k}$$

$$\frac{3}{4} = e^{-2k}$$

$$\ln\left(\frac{3}{4}\right) = -2k$$

$$k = -\frac{1}{2}\ln\left(\frac{3}{4}\right)$$

Plug k into C.

$$C = 40000\exp\left[-5\left(-\frac{1}{2}\ln\left(\frac{3}{4}\right)\right)\right]$$

$$= 40000\exp\left[\frac{5}{2}\ln\left(\frac{3}{4}\right)\right]$$

$$= 40000\,\cancel{\exp}\left[\cancel{\ln}\left(\frac{3}{4}\right)^{5/2}\right]$$

$$= 40000\left(\frac{3}{4}\right)^{5/2}$$

$$P(10) = 40000\left(\frac{3}{4}\right)^{5/2} e^{\left[-\frac{1}{2}\ln\left(\frac{3}{4}\right)\times 10\right]}$$

Riemann Sums:

$f(x) = \ln(2x+3)$

$n = 10$

$[0, 10]$

Left: $\sum_{i=0}^{n-1} f(x_i)\Delta x$

Right: $\sum_{i=1}^{n} f(x_i)\Delta x$

① $\Delta x = \frac{10-0}{10} = 1$

② $x_i = a + i\Delta x = 0 + i \cdot 1 = i$

③ $f(x_i) = \ln(2i+3)$

$\text{Left} = \sum_{i=0}^{9} \ln(2i+3) \cdot 1$

$\text{Right} = \sum_{i=1}^{10} \ln(2i+3)$

Estimate the difference of Right & Left Sum

$$L_{10} = \sum_{i=0}^{9} a_i \qquad R_{10} = \sum_{i=1}^{10} a_i$$

$$
\begin{aligned}
R_{10} &= \phantom{a_0 +{}} \cancel{a_1} + \cancel{a_2} + \cdots + \cancel{a_8} + \cancel{a_9} + a_{10} \\
-\big(L_{10} &= a_0 + \cancel{a_1} + \cancel{a_2} + \cdots + \cancel{a_8} + \cancel{a_9} \big)
\end{aligned}
$$

$$R_{10} - L_{10} = a_{10} - a_0$$

Look back to the previous question

$$R_{10} - L_{10} = \ln(2(10)+3) - \ln(2(0)+3)$$

Trapezoid Rule

$f(x) = x^2 + 1$

$n = 4$

$[1, 9]$

① $\Delta x = \frac{9-1}{4} = 2$

② $x_0 = 1$ $\quad f(x_0) = 2$

$x_1 = 3$ $\quad f(x_1) = 10$

$x_2 = 5$ $\quad f(x_2) = 26$

$x_3 = 7$ $\quad f(x_3) = 50$

$x_4 = 9$ $\quad f(x_5) = 82$

$$\boxed{\begin{array}{l} f(x_0) = 2 \\ 2f(x_1) = 20 \\ 2f(x_2) = 52 \\ 2f(x_3) = 100 \\ f(x_5) = 82 \end{array}}$$

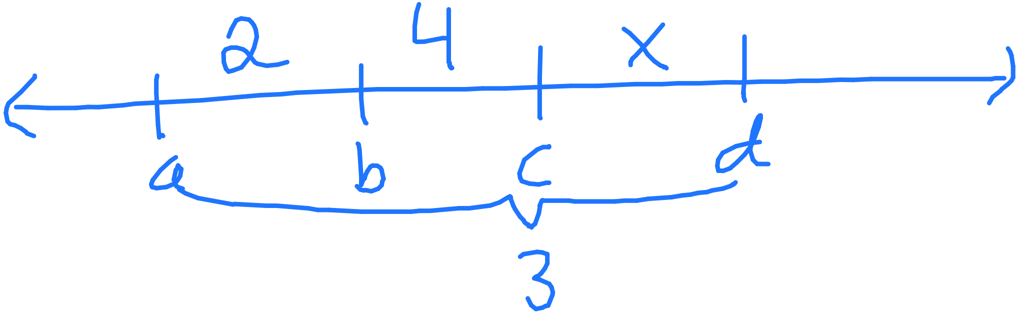
Sum

$$T_4 = \frac{1}{2} \Delta x \cdot \text{Sum}$$

$$= \frac{1}{2} \cdot 2\,(2 + 20 + 52 + 100 + 82)$$

Definite Integrals: $\int_a^b f(x)\,dx = 2$, $\int_a^d f(x)\,dx = 3$

$\int_b^c f(x) = 4$. Find $\int_c^d f(x)\,dx$

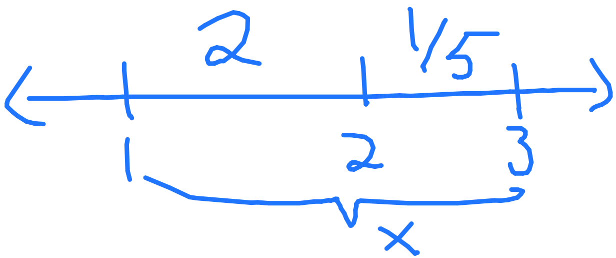


$$2+4+x=3 \rightarrow 6+x=3 \rightarrow x=-3=\int_c^d f(x)$$

$\int_2^1 f(x)\,dx = -2$, $\int_2^3 5f(x)\,dx = 1$. Find $\int_1^3 f(x)\,dx$.

$$-\int_1^2 f(x)\,dx = -2 \qquad 5\int_2^3 f(x)\,dx = 1$$

$$\int_1^2 f(x)\,dx = 2 \qquad \int_2^3 f(x)\,dx = \frac{1}{5}$$



$$\int_1^3 f(x)\,dx = \frac{11}{5}$$

Exponential Growth/Decay

$$\frac{dy}{dt} = ky \Rightarrow y = Ce^{kt}$$

half-life $\rightarrow$ $k = \dfrac{\ln(1/2)}{\text{half-life}} = \dfrac{-\ln(2)}{\text{half-life}}$

$\dfrac{\ln(1/2)}{\text{half-life}}$ $\downarrow$ Multiple choice Answer

$\dfrac{-\ln(2)}{\text{half-life}}$ $\downarrow$ Easier to plug into calculator

$$\ln\left(\frac{1}{2}\right) = \ln(2^{-1})$$
$$= -\ln 2$$

Say $k=2$, $C=10$. Find time when $y=20$

$$y=10e^{2t}$$

$$20=10e^{2t}$$

$$2=e^{2t}$$

$$\ln 2=2t$$

$$\frac{1}{2}\ln 2=t$$