

## Differentiation Rules

## Integration Rules

$$\frac{d}{dx}(c) = 0$$

$$\int 0 dx = c$$

$$\frac{d}{dx}(kx) = k$$

$$\int k dx = kx + c$$

$$\frac{d}{dx}(kf(x)) = kf'(x)$$

$$\int kf'(x) dx = kF(x) + c$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \text{ when } n \neq -1$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\int \cos x dx = \sin x + c$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\int \sin x dx = -\cos x + c$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\int \sec^2 x dx = \tan x + c$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\int \csc^2 x dx = -\cot x + c$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\int \sec x \tan x dx = \sec x + c$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\int \csc x \cot x dx = -\csc x + c$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\int e^x dx = e^x + c$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}, x > 0$$

$$\int \frac{1}{x} dx = \ln|x| + c$$

- For Exam 4, we can ignore the +c for the integration rules because we have upper and lower bound.

$$\int_a^b f(x) dx = \text{Signed Area}$$

- Always take derivative of your answer, before evaluating when integrating. Especially when you have trig functions!!!

Signed Area: Area enclosed by  $f(x)$  and  $x$ -axis.

- If  $f(x)$  above the  $x$ -axis, then area is positive.
- If  $f(x)$  below the  $x$ -axis, then area is negative.

## Riemann Sums

Left:  $L_n = \sum_{i=0}^{n-1} f(x_i) \Delta x$       Right:  $R_n = \sum_{i=1}^n f(x_i) \Delta x$

where  $x_i = a + i\Delta x$  and  $\Delta x = \frac{b-a}{n}$

Tips: ① Determine what  $a$ ,  $b$ , and  $n$  are.

② Calculate the following in the following order:

- ①  $\Delta x$       ②  $x_i$       ③  $f(x_i)$       ④  $L_n$  or  $R_n$

## Trapezoid Rule

$$T_n = \frac{1}{2} \Delta x (f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n))$$

where  $x_i = a + i\Delta x$  and  $\Delta x = \frac{b-a}{n}$

Tips: ① Determine what  $a$ ,  $b$ , and  $n$  are.

② Calculate the following in the following order:

- |              |           |              |               |
|--------------|-----------|--------------|---------------|
| ① $\Delta x$ | ② $x_0$   | ③ $f(x_0)$   | ④ $f(x_0)$    |
|              | $x_1$     | $f(x_1)$     | $2f(x_1)$     |
|              | $\vdots$  | $\vdots$     | $\vdots$      |
|              | $x_{n-1}$ | $f(x_{n-1})$ | $2f(x_{n-1})$ |
|              | $x_n$     | $f(x_n)$     | $f(x_n)$      |

③ Sum all values from ④ and multiply by  $\frac{1}{2}\Delta x$   
Yielding  $T_n$ .

When given the graph of a shaded region, to determine the integral,

① Left-most x-value  $\Rightarrow a$

② Right-most x-value  $\Rightarrow b$

③ the line connecting them  $\Rightarrow f(x)$

$\hookrightarrow$  Use algebra to determine the equation

$$\int_a^b f(x) dx$$

### Properties of Definite Integrals

①  $\int_a^a f(x) dx = 0$       ②  $\int_a^b f(x) dx = -\int_b^a f(x) dx$

③  $\int_a^b k f(x) dx = k \int_a^b f(x) dx$

④  $\int_a^b [f(x) \pm g(x)] dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$

⑤  $\int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx$

FTC:  $\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$

### Exponential Growth/Decay Models

$\frac{dy}{dt} = y' = ky \Rightarrow y = Ce^{kt}$  where  $k$  - growth rate } Constant  
 $C$  - initial value }

Growth  $\Rightarrow k > 0$

Decay  $\Rightarrow k < 0$

Half-Life  $\Rightarrow k = -\frac{\ln(2)}{\text{half-life}}$  or  $k = \frac{\ln(1/2)}{\text{half-life}}$