MA 16010: Applied Calculus 1 Extra Credit 3 (Solutions)

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:_____

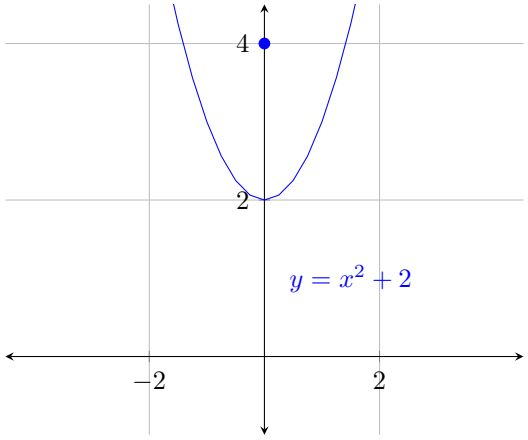
1. Find the points on the curve $y = x^2 + 2$ closest to the point $(0, 4)$.

To receive full credit for this problem, you must show all 7 steps, as discussed in lecture.

Solution: Steps:

(1) Distance

(2) Graph



(3) $D = (x-0)^2 + (y-4)^2 = x^2 + (y-4)^2$

(4) $y = x^2 + 2$

(5) Domain of x: $(-\infty, \infty)$
Domain of y: $[2, \infty)$

(6) Plug (4) into (3).

$$\begin{aligned} D(x,y) &= x^2 + (y-4)^2 \\ D(x) &= x^2 + (x^2+2-4)^2 \\ &= x^2 + (x^2-2)^2 \\ &= x^2 + x^4 - 4x^2 + 4 \\ &= x^4 - 3x^2 + 4 \end{aligned}$$

Take the derivative and set $= 0$.

$$\begin{aligned} D'(x) &= 4x^3 - 6x = 0 \\ &= 2x(2x^2 - 3) = 0 \end{aligned}$$

Hence $x = 0, \pm\sqrt{3/2}$.

Now we need to check for absolute minimum. Since we have 3 critical numbers, plug each into D and whichever is the smallest is the absolute minimum.

x	$y = x^2 + 2$	$D = x^2 + (y-4)^2$	Conclusion
$-\sqrt{\frac{3}{2}}$	$\frac{7}{2}$	$\frac{7}{4}$	Absolute Min
0	2	4	
$\sqrt{\frac{3}{2}}$	$\frac{7}{2}$	$\frac{7}{4}$	Absolute Min

(7) Answer: $\boxed{\left(-\sqrt{\frac{3}{2}}, \frac{7}{2}\right)}$ and $\boxed{\left(\sqrt{\frac{3}{2}}, \frac{7}{2}\right)}$