8 pt When computing limits numerically, which of the following will lead to the conclusion that $\lim_{x\to 5} f(x) = 0$?

1. ~~**A**~~

x	-0.1	-0.01	-0.001	-0.0001	0	0.0001	0.001	0.01	0.1
$f(x)$	4.9	4.09	4.009	4.0009		5.0001	5.001	5.01	5.1

B

x	4.9	4.99	4.999	4.9999	5	5.0001	5.001	5.01	5.1
$f(x)$	-0.1	-0.01	-0.001	-0.0001		0.0001	0.001	0.01	0.1

C

x	4.9	4.99	4.999	4.9999	5	5.0001	5.001	5.01	5.1
$f(x)$	1	1	1	1		0.0001	0.001	0.01	0.1

~~**D**~~

x	-0.1	-0.01	-0.001	-0.0001	0	0.0001	0.001	0.01	0.1
$f(x)$	-0.1	-0.01	-0.001	-0.0001		0.0001	0.001	0.01	0.1

~~**E**~~

x	-0.1	-0.01	-0.001	-0.0001	0	0.0001	0.001	0.01	0.1
$f(x)$	4.9	4.99	4.999	4.9999		5	5	5	5

F

x	4.9	4.99	4.999	4.9999	5	5.0001	5.001	5.01	5.1
$f(x)$	4.9	4.99	4.999	4.9999		5.0001	5.001	5.01	5.1

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8 pt Given

$$f(x) = \frac{x-4}{x^2-4x}. = \frac{x-4}{x(x-4)} = \frac{1}{x}$$

Choose the correct statement regarding $\lim_{x\to 0} f(x)$ and $\lim_{x\to 4} f(x)$.

2. ~~**A**~~ $\lim_{x\to 0} f(x) = 1$; $\lim_{x\to 4} f(x) = 0$

~~**B**~~ $\lim_{x\to 0} f(x) = -4$; $\lim_{x\to 4} f(x) = 0$

C $\lim_{x\to 0} f(x)$ does not exist; $\lim_{x\to 4} f(x) = \frac{1}{4}$

D $\lim_{x\to 0} f(x)$ does not exist; $\lim_{x\to 4} f(x)$ does not exist.

~~**E**~~ $\lim_{x\to 0} f(x) = 1$; $\lim_{x\to 4} f(x)$ does not exist.

~~**F**~~ $\lim_{x\to 0} f(x) = -4$; $\lim_{x\to 4} f(x) = \frac{1}{4}$

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8 pt Let

$$f(x) = \begin{cases} 5x, & x < 1 \\ 20 - x, & 1 \leq x < 3 \\ 2x^2 - 1, & x \geq 3 \end{cases}$$

$\lim\limits_{x \to 1^-} 5x = 5$

$\lim\limits_{x \to 1^+} (20-x) = 19$

$\lim\limits_{x \to 3^-} (20-x) = 17$

$\lim\limits_{x \to 3^+} (2x^2-1) = 17$

Which statement is true about the discontinuities of $f(x)$?

3. A $f(x)$ is only discontinuous at $x = 3$.

B $f(x)$ is only discontinuous at $x = 5$.

C $f(x)$ is only discontinuous at $x = 19$.

D $f(x)$ is only discontinuous at $x = 1$.

E $f(x)$ is only discontinuous at $x = 17$.

F $f(x)$ does not have any discontinuities.

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8 pt Given $f(x) = \dfrac{x^2 - 36}{x^2 - 7x + 6}$. Which of the following are true?

Hole @ x=6

$f(x) = \dfrac{x^2-36}{x^2-7x+6} = \dfrac{\cancel{(x-6)}(x+6)}{\cancel{(x-6)}(x-1)}$

VA @ x=1

I. $f(x)$ has a hole at $x = 6$.

II. $f(x)$ has a hole at $x = 1$.

III. $f(x)$ has a hole at $x = -6$.

IV. $f(x)$ has a vertical asymptote at $x = 6$.

V. $f(x)$ has a vertical asymptote at $x = 1$.

VI. $f(x)$ has a vertical asymptote at $x = -6$.

4. A II and III

B I and V

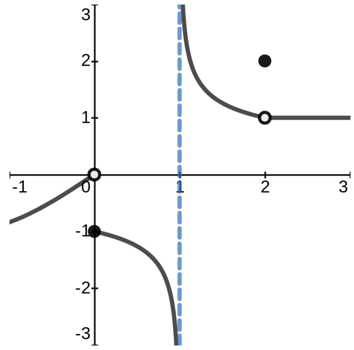
C I and II

D IV and V

E II and IV

F I and VI

8 pt The following is the graph of $f(x)$:



Which of the following statements is/are TRUE?

I. $\lim_{x \to 2} f(x) = 2$

II. $\lim_{x \to 1} f(x) = \infty$

III. $\lim_{x \to 0^-} f(x) = 0$

5. A Only II is true.

B Only I and III are true.

C Only I and II are true.

D Only I is true.

E Only II and III are true.

F Only III is true.

8 pt Jim is deriving the derivative of $f(x)$ using the limit definition and he writes, correctly,

$$f'(x) = \lim_{h\to 0} \frac{5(x+h)^2 - 5x^2}{h}.$$

Which of the following functions could be $f(x)$?

6. A $5x^2 - x^5$

B $5x$

C $x^2 - 5x$

D $x^5 - 5x^2$

E x^2

$\boxed{\textbf{F} \quad 5x^2}$

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8 pt Find the derivative of $y = \frac{1}{9x^4} - \frac{1}{4}\sqrt{x}$. $= \frac{1}{9} \cdot \frac{1}{x^4} - \frac{1}{4}x^{1/2}$

$= \frac{1}{9}x^{-4} - \frac{1}{4}x^{1/2}$

$y' = -\frac{4}{9}x^{-4-1} - \frac{1}{4} \cdot \frac{1}{2}x^{-1/2}$

$= -\frac{4}{9}x^{-5} - \frac{1}{8}x^{-1/2}$

$= -\frac{4}{9x^5} - \frac{1}{8\sqrt{x}}$

7. $\boxed{\textbf{A} \quad -\frac{4}{9x^5} - \frac{1}{8\sqrt{x}}}$

B $\frac{9}{4}x^3 - \frac{1}{4\sqrt{x}}$

C $\frac{1}{36x^3} - \frac{\sqrt{x^3}}{8}$

D $\frac{9}{4}x^3 - \frac{\sqrt{x^3}}{8}$

E $\frac{1}{36x^3} - \frac{1}{8\sqrt{x}}$

F $-\frac{4}{9x^5} - \frac{1}{4\sqrt{x}}$

$\boxed{8\ pt}$ Find the equation of the tangent line to the graph of $y = \sqrt{\frac{1}{x}} - 1$ at $x = 4$.

8. A $y = -4x + \frac{33}{2}$

$\boxed{\textbf{B} \quad y = -\frac{1}{16}x - \frac{1}{4}}$

C $y = -4x + \frac{31}{2}$

D $y = \frac{1}{2}x - \frac{3}{2}$

E $y = -\frac{1}{16}x + \frac{3}{4}$

F $y = \frac{1}{2}x - \frac{5}{2}$

$y = \frac{1}{\sqrt{x}} - 1 = x^{-1/2} - 1 \Rightarrow y = \frac{1}{\sqrt{4}} - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$

$y' = -\frac{1}{2}x^{-3/2} = -\frac{1}{2(\sqrt{x})^3} \Rightarrow y'(4) = -\frac{1}{2(\sqrt{4})^3} = \frac{-1}{2 \cdot 2^3} = \frac{-1}{16}$

$y - (-\frac{1}{2}) = -\frac{1}{16}(x-4)$

$y + \frac{1}{2} = -\frac{1}{16}x + \frac{1}{4}$

$-\frac{1}{2} \qquad -\frac{1}{2}$

$y = -\frac{1}{16}x - \frac{1}{4}$

$\boxed{8\ pt}$ The population of a city since 1990 can be modeled as

$$P(t) = t^2 + 1234t + 116000,$$

where $t = 0$ corresponds to the year 1990. In which year is the population increasing at the rate of 1306 people per year?

9. A 2017

$\boxed{\textbf{B} \quad 2026}$

C 1995

D 1998

E 2043

F 2036

Solve $P'(t) = 1306$.

$P'(t) = 2t + 1234 = 1306$

$\frac{2t}{2} = \frac{72}{2}$

$t = 36$

$t = 0 \Rightarrow 1990$

$t = 36 \Rightarrow 1990 + 36 = 2026$

$\boxed{8\ pt}$ Given $f(x) = (x^2 - 3x)(e^x + 2)$. Find $f'(0)$.

10. A -6

B 4

C 12

D -3

$\boxed{\textbf{E} \quad -9}$

F 0

$u = x^2 - 3x \qquad v = e^x + 2$

$u' = 2x - 3 \qquad v' = e^x$

$f' = u'v + v'u$

$= (2x-3)(e^x+2) + e^x(x^2-3x)$

$f'(0) = (-3)(1+2) + 1(0)$

$= -9$

$\boxed{8\ pt}$ Find $f'(1)$ given

$$f(x) = \frac{x^2 + x}{x^2 + 1}.$$

$u(x) = x^2 + x \quad v(x) = x^2 + 1$

$u'(x) = 2x + 1 \quad v'(x) = 2x$

$f' = \frac{u'v - v'u}{v^2} = \frac{(2x+1)(x^2+1) - 2x(x^2+x)}{(x^2+1)^2}$

$f'(1) = \frac{3 \cdot 2 - 2 \cdot 2}{2^2} = \frac{2}{4} = \frac{1}{2}$

11. A 3

B $\frac{2}{5}$

C 1

D $\frac{5}{2}$

E $\frac{3}{2}$

$\boxed{\textbf{F} \quad \frac{1}{2}}$

$\boxed{8\ pt}$ If $h(x) = 2\tan x - \frac{1}{2}\cot x$, then $h'(\frac{\pi}{4}) =$

$h'(x) = 2\sec^2 x - \frac{1}{2}(-\csc^2 x)$

$= 2\sec^2 x + \frac{1}{2}\csc^2 x$

$h'(\frac{\pi}{4}) = 2\sec^2(\frac{\pi}{4}) + \frac{1}{2}\csc^2(\frac{\pi}{4})$

$= 2 \cdot 2 + \frac{1}{2} \cdot 2$

$= 5$

12. A 3

B 8

$\boxed{\textbf{C} \quad 5}$

D 4

E 1

F 10

