Formulas

Product Rule: $f(x) = u(x)v(x)$

$$f'(x) = u'(x)v(x) + v'(x)u(x)$$

Quotient Rule: $f(x) = \dfrac{u(x)}{v(x)}$

$$f'(x) = \frac{u'(x)v(x) - v'(x)u(x)}{v^2(x)}$$

Power Rule: $f(x) = x^n$ where n is a #

$$f'(x) = nx^{n-1}$$

Tangent Line:
1) Find y'
2) Plug $x=c$ for y, and y'.
3) Plug values from 2 into
$$y - y(c) = y'(c)(x-c)$$
4) Solve for y.

Derivatives of Trig Functions

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

Recipe for Solving a Related Rates Problem

Step 1: Draw a good picture. Label all constant values and give variable names to any changing quantities.

Step 2: Determine what information you **KNOW** and what you **WANT** to find.

Step 3: Find an equation relating the relevant variables. This usually involves a formula from geometry, similar triangles, the Pythagorean Theorem, or a formula from trigonometry. Use your picture!

Step 4: Use implicit differentiation to differentiate the equation with respect to time t.

Step 5: Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you **WANT**. Do **NOT** substitute before this step!

Some Useful Formulas

Right Triangle
Pythagorean Theorem:
$a^2 + b^2 = c^2$

Triangle
$A = \frac{1}{2} bh$
$P = a + b + c$

Equilateral Triangle
$h = \frac{\sqrt{3}}{2} s$ $\quad A = \frac{\sqrt{3}}{4} s^2$

Rectangle
$A = lw$
$P = 2l + 2w$

Trapezoid
$A = \frac{1}{2}(a + b)h$

Circle
$A = \pi r^2$ $\quad C = 2\pi r$

Circular Sector
$A = \frac{1}{2} r^2 \theta$ $\quad s = r\theta$

Circular Ring
$A = \pi(R^2 - r^2)$

Rectangular Box
$V = lwh$
$S = 2(hl + lw + hw)$

Sphere
$V = \frac{4}{3}\pi r^3$ $\quad S = 4\pi r^2$

Right Circular Cylinder
$V = \pi r^2 h$ $\quad S = 2\pi rh$

Right Circular Cone
$V = \frac{1}{3}\pi r^2 h$

E2P1

Position/Velocity/Acceleration

Position	$s(t)$
Velocity	$v(t) = s'(t)$
Acceleration	$a(t) = v'(t) = s''(t)$

Chain Rule:

$f(x) = a(b(x))$

$f'(x) = a'(b(x))\, b'(x)$

Derivatives of Exponential & Logarithmic Functions

$\frac{d}{dx}[e^x] = e^x$ $\qquad$ $\frac{d}{dx}[\ln x] = \frac{1}{x}$

Higher Order Derivatives

Idea: Take the derivative of the derivative.

Implicit Differentiation

Idea: Take the derivative of each side. Using $\frac{d}{dx}$.

Solve for $\frac{dy}{dx}$.

Useful Formulas

$\frac{d}{dx}(xy) = y + x\frac{dy}{dx}$ $\qquad$ $\frac{d}{dx}\left(\frac{x}{y}\right) = \frac{y - x\frac{dy}{dx}}{y^2} = \frac{1}{y} - \frac{x}{y^2}\frac{dy}{dx}$

Critical Numbers:

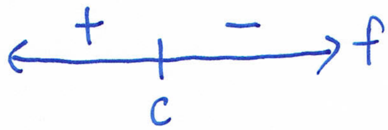
1. Find the domain of y
2. Find out when $f'(x) = 0$ or $f'(x)$ is undefined.
3. Check that the x-values found in 2 don't match 1.

If they do, then that value isn't a critical number.

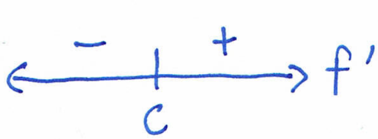
F2P2

Increasing if $f'(x) > 0$ on an interval
Decreasing if $f'(x) < 0$ on an interval

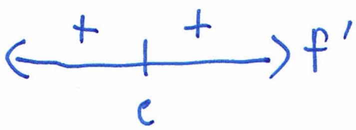
The First Derivative Test: c is a critical #.

① 

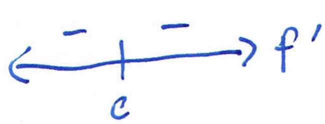
ex.  $\Rightarrow$ rel max @ $x = c$

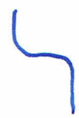
② 

ex.  $\Rightarrow$ rel min @ $x = c$

③ 

ex.  $\Rightarrow$ neither

④ 

ex.  $\Rightarrow$ neither

How to Determine Increasing/Decreasing & Relative Extrema

① Find critical pts.
② Draw a # line with the points from ①.
③ Determine test pts using ②.
④ Plug those values into f' to determine whether its positive or negative.
We don't actually care about the value. Only whether it is + or −.
⑤ Use definition of increasing/decreasing and the First Derivative Test to determine
ⓐ Increasing/Decreasing
ⓑ Relative Extrema

E2P3