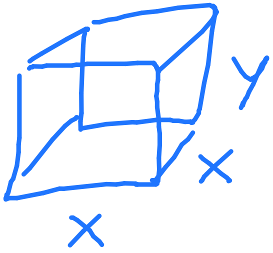
Quiz 11: Box square base, open top

Material = 100. Max Volume.



$V = x^2 y$

$100 = A = x^2 + 4xy$

$y = \frac{100 - x^2}{4x}$

$V = x^2\left(\frac{100 - x^2}{4x}\right)$

$V = \frac{x^{\cancel{2}}}{4\cancel{x}}(100 - x^2)$

$V = \frac{1}{4}(100x - x^3)$

$V' = \frac{1}{4}(100 - 3x^2)$

$= 0$

$x = \sqrt{\frac{100}{3}}$

Plug in y

Plug x in V

Asymptotes

Vertical $\Rightarrow$ Denominator once f(x) is simplify

ex. $f(x) = \dfrac{x-2}{x^2-4} = \dfrac{\cancel{x-2}}{\cancel{(x-2)}(x+2)}$

VA @ $x=-2$ Hole @ $x=2$

Domain: $(-\infty,-2) \cup (-2,2) \cup (2,\infty)$

Slant or Horizontal Never Both

ex. $f(x) = \frac{x+2}{2x+3} \Rightarrow$ HA @ $y = \frac{1}{2}$

ex. $g(x) = \frac{x^{3}+2}{2x} \Rightarrow$ None $3-1=2$

Difference $= 0 \Rightarrow$ HA

$>1 \Rightarrow$ None

Difference $= 1 \Rightarrow$ Slant

ex. $h(x) = \dfrac{2x^2+3x+1}{x-2}$ $\Rightarrow$ Slant

$$\begin{array}{c|ccc} 2 & 2 & 3 & 1 \\ & \downarrow & 4 & 14 \\ \hline & 2 & 7 & 15 \end{array}$$

$\Rightarrow$ Slant @ $y = 2x+7$

ex. $h(x) = \dfrac{2x^3+3x+1}{x^2+2}$ $\Rightarrow$ Long Division

Limits @ ∞

$$\lim_{x\to\infty} \frac{x^2+2x+3}{2x+1} \Rightarrow \lim_{x\to\infty} \frac{x^2}{2x} = \lim_{x\to\infty} \frac{\cancel{x}}{2} = \infty$$

$$\lim_{x\to\infty} \frac{x}{2} - \frac{6}{x} = \lim_{x\to\infty} \frac{x^2-12}{2x} = \infty$$

$$\lim_{x\to-\infty} \qquad \downarrow \quad = -\infty$$

Integration

$$\int \frac{x^2+\sqrt{x}}{x}\,dx = \int \frac{x^2}{x} + \frac{x^{1/2}}{x}\,dx = \int x + x^{-1/2}\,dx$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$= \frac{x^2}{2} + \frac{x^{1/2}}{1/2} + C$$

$$= \frac{x^2}{2} + 2x^{1/2} + C$$

$$\boxed{\int \frac{1}{x}\,dx = \ln|x| + c}$$

$$\int \frac{1}{x}\,dx = \int x^{-1}\,dx = \cancel{\frac{x^{-1+1}}{-1+1} + C = \frac{x^{0}}{0} + C}$$

$$\downarrow$$

$$= \ln|x| + C$$

$$\int \frac{1}{x^{2}}\,dx = \int x^{-2}\,dx = \frac{x^{-2+1}}{-2+1} = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$$

$$\int f(x)\,dx = F(x)$$

$$f(x) = F'(x)$$

A

B

C

D

E

<u>PE #3:</u> $f(x) = \dfrac{4x}{x^2-4} \Rightarrow$ VA: $x = \pm 2$

$\cancel{A}$ B C D $\cancel{E}$

y-intercept: $(0,0)$

$$f'(x) = \frac{-4(x^2+4)}{(x^2-4)^2} = 0 \Rightarrow f'(x) \neq 0$$

$-$ $\quad$ $-$ $\quad$ $-$

-3 $\quad$ 0 $\quad$ 3

$\longleftarrow$ -2 $\quad$ 2 $\longrightarrow$ f'

Antiderivative

$$\int \tan x \cos x + x\, dx = \int \frac{\sin x}{\cancel{\cos x}} \cdot \cancel{\cos x} + x\, dx$$

$$= \int \sin x + x\, dx$$

$$= -\cos x + \frac{x^2}{2} + C$$

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$$

HW 28.7:

IVP

$$\begin{cases} \dfrac{dP}{dt} = \cancel{9\sqrt{t}} = 9t^{1/2} \\ P(0) = 700 \end{cases}$$

$P(7) = ?$

$$\int \frac{dP}{\cancel{dt}} \cancel{dt} = \int 9t^{1/2}\,dt$$

$$P = 9\frac{t^{3/2}}{3/2} + C$$

$$P = 9 \cdot \frac{2}{3} t^{3/2} + C = 6t^{3/2} + C$$

$$700 = 6(0)^{3/2} + C$$

$$700 = C$$

$$P = 6t^{3/2} + 700$$

$$P(7) = 6(7)^{3/2} + 700$$

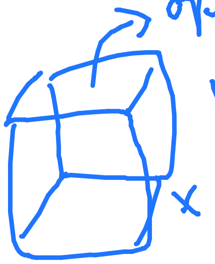
Limits @ ∞

$$\lim_{x\to\infty} \frac{2x^2+2x+1}{x^2+1} \Rightarrow \lim_{x\to\infty} \frac{2x^2}{x^2} = \lim_{x\to\infty} 2 = 2$$

$$\lim_{x\to-\infty} \frac{2x^2+3x}{1-x} \Rightarrow \lim_{x\to-\infty} \frac{2x^2}{-x} = \lim_{x\to-\infty} -2x = \infty$$

$$\lim_{x\to\infty} \frac{6}{x} - \frac{x}{2} = \lim_{x\to\infty} \frac{12-x^2}{2x} \Rightarrow \lim_{x\to\infty} \frac{-x^2}{2x} = \lim_{x\to\infty} \frac{-x}{2} = -\infty$$

Optimization: Q11



Maximize V.

$$V = x^2 y$$

$$100 = A = x^2 + 4xy$$

$$y = \frac{100 - x^2}{4x}$$

$$V = x^2\left(\frac{100 - x^2}{4x}\right)$$

$$= \frac{x^2}{4x}(100 - x^2)$$

$$= \frac{x}{4}(100 - x^2)$$

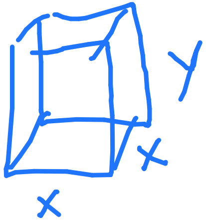
$$= \frac{1}{4}(100x - x^3)$$

$$V' = \frac{1}{4}(100 - 3x^2) = 0$$

$$x = \sqrt{\frac{100}{5}}$$

Plug x into V.

PE #22:



$$16 = V = x^2 y$$

$$C = \$4(4xy) + \$1(x^2 + x^2)$$

$$= 16xy + 2x^2$$

$$16 = x^2 y \Leftrightarrow y = \frac{16}{x^2}$$

$$C = 16x\left(\frac{16}{x^2}\right) + 2x^2$$

$$= \frac{256}{x} + 2x^2$$

$$= 256x^{-1} + 2x^2$$

$$C' = -256x^{-2} + 4x = 0$$

$$\frac{-256}{x^2} + 4x = 0$$

$$\frac{4x}{1} = \frac{256}{x^2} \Rightarrow x^3 = 64$$

$$x = 4$$

HW 27: 9-11

9 $\int x^2 \sin(3x)\,dx$

~~A~~
~~B~~
~~C~~
~~D~~
$\boxed{E}$

Find derivative of each and match with $x^2 \sin(3x)$

$\int f(x)\,dx = F(x)$

$\Longleftrightarrow$

$f(x) = F'(x)$

HW 28.8:

$$\begin{cases} a(t) = -32 \\ v(0) = 14 \\ s(0) = 800 \end{cases} \Longleftrightarrow \begin{cases} s''(t) = -32 \checkmark \\ s'(0) = 14 \checkmark \\ s(0) = 800 \end{cases}$$

① t when $s(t) = 0$

$$\int s''(t)\,dt = \int -32\,dt$$

$$s'(t) = -32t + C$$

$$14 = -32(0) + C$$

$$14 = C$$

$$s'(t) = -32t + 14$$

$$\int s'(t)\,dt = \int -32t + 14\,dt$$

$$s(t) = -\frac{32t^2}{2} + 14t + C$$

$$= -16t^2 + 14t + C$$

$$800 = s(0) = 0 + 0 + C$$

$$800 = s(0)$$

$$s(t) = -16t^2 + 14t + 800$$

Last part $s(t) = 0$.

$$-2(\underbrace{8t^2-7t-400}) = 0$$

$$t = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(8)(-400)}}{2(4)}$$

Use positive one.

② Answer from ① plug $v(t)$.

HW 28.6:

$$\begin{cases} y'' = 6\sin(x) + \frac{6}{\pi^2}x \\ y'(\pi) = 6 \\ y(\pi/2) = -6 \end{cases}$$

$$\int y'' dy = \int 6\sin x + \frac{6}{\pi^2}x \, dx$$

$$y' = -6\cos x + \frac{6}{\pi^2} \cdot \frac{x^2}{2} + C$$

$$= -6\cos x + \frac{3}{\pi^2} \cdot x^2 + C$$

$$6 = -6\cos\pi + \frac{3}{\cancel{\pi^2}}\cancel{\pi^2} + C$$

$$6 = 6 + 3 + C$$

$$-3 = C$$

$$y' = -6\cos x + \frac{3}{\pi^2}x^2 - 3$$

$$\int y'\,dx = \int -6\cos x + \frac{3}{\pi^2}x^2 - 3 \; dx$$

$$y = -6\sin x + \frac{3}{\pi^2}\cdot\frac{x^3}{3} - 3x + C$$

$$y\left(\frac{\pi}{2}\right) = -6$$

$$-6 = -6\sin\left(\frac{\pi}{2}\right) + \frac{\cancel{3}}{\pi^2}\cdot\frac{(\pi/2)^3}{\cancel{3}} - 3\frac{\pi}{2} + C$$

$$-6 = -6 + \frac{1}{\cancel{\pi^2}}\cdot\frac{\pi^{\cancel{3}}}{8} - \frac{3\pi}{2} + C$$

$$0 = \frac{\pi}{8} - \frac{3\pi}{2} + C$$

$$\frac{11\pi}{8} = C$$

$$y(x) = -6\sin x + \frac{x^3}{\pi^2} - 3x + \frac{11\pi}{8}$$