8 pt Find the x value at which the inflection point of $g(x) = 10x^3 + 30x^2 + 20$ occurs.

$g'(x) = 30x^2 + 60x$

$g''(x) = 60x + 60 = 0$

$60(x+1) = 0$

$x = -1$

$-$ $+$ f''

-1

Inflection Pt

1. A 0

B $\frac{1}{2}$

$\boxed{\textbf{C} \quad -1}$

D -2

E 1

F 2

---

8 pt Which of the following limits equals ∞ ?

2. ~~**A**~~ $\lim_{x\to\infty} \frac{x-2}{x^2+x+1} = \lim_{x\to\infty} \frac{x}{x^2} = \lim_{x\to\infty} \frac{1}{x} = 0$

$\boxed{\textbf{B} \quad \lim_{x\to\infty} \frac{2x^4 - x + 1}{x^2 - 8}} = \lim_{x\to\infty} \frac{2x^4}{x^2} = \lim_{x\to\infty} 2x^2 = \infty$

C $\lim_{x\to\infty} \frac{2x+1}{5x-10}$

D $\lim_{x\to\infty} \frac{-x^2+1}{x-5}$

E $\lim_{x\to\infty} \frac{5x^2+9}{6x^2-2x+5}$

F $\lim_{x\to\infty} \frac{-x^3+3x}{4x^4+3x^3+2}$

---

8 pt Find the largest open interval where the function $f(x) = \frac{1}{12}x^4 + \frac{1}{3}x^3 + 9x$ is concave down.

3. A $(0, \infty)$

B $(-\infty, 0)$

$\boxed{\textbf{C} \quad (-2, 0)}$

D $(2, \infty)$

E $(-\infty, -2)$

F $(0, 2)$

$f'(x) = \frac{4}{12}x^3 + \frac{3}{3}x^2 + 9$

$= \frac{1}{3}x^3 + x^2 + 9$

$f''(x) = \frac{3}{3}x^2 + 2x$

$= x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0, -2$

$+$ $-$ $+$

-3 -1 1 f''

-2 0

$\Rightarrow$ Concave Down

$(-2, 0)$

8 pt Find the x value at which the absolute maximum of $y = x^3 - 12x$ occurs on the interval of $[-3, 3]$.

$y' = 3x^2 - 12 = 0$
$3(x^2 - 4) = 0$
$3(x-2)(x+2) = 0$
$x = \pm 2$

x	$y = x^3 - 12x$	Conclusion
-3	$-27+36 = 9$	
-2	$-8+24 = 16$	max
2	$8-24 = -16$	
3	$27-36 = -9$	

4. **A** 1

 B -3

 C 0

 D 3

 E 2

 $\boxed{\textbf{F} \quad -2}$

---

8 pt Choose the number of true statements regarding the function $f(x) = \frac{x^2 - 1}{x - 4}$. $= \frac{(x-1)(x+1)}{x-4}$

I. The y-intercept is $(0, \frac{1}{4})$.
II. The x-intercepts are $(-1, 0)$ and $(1, 0)$.
III. $f(x)$ has one vertical asymptote.
IV. $f(x)$ does not have any horizontal asymptote.
V. $f(x)$ has one slant asymptote.

y-intercept
when $x = 0$, $y = \frac{0-1}{0-4} = \frac{1}{4}$

x-intercept
when $y = 0$,
$0 = \frac{x^2-1}{x-4}$
$0 = x^2 - 1$
$1 = x^2$
$x = \pm 1$

Slant/Horizontal
Difference of power $= 1$
$\Rightarrow$ Slant

VA: $0 = x - 4$
$x = 4$

5. **A** No statement is true.

 B Only one statement is true.

 C Only two statements are true.

 D Only three statements are true.

 E Only four statements are true.

 $\boxed{\textbf{F} \quad \text{All five statements are true.}}$

$\boxed{8\ pt}$ Evaluate

$$\int \frac{6x^2+1}{8x}\,\mathrm{d}x.$$

$$= \int \frac{6x^2}{8x} + \frac{1}{8x}\,dx$$

$$= \int \frac{6}{8}x + \frac{1}{8}\cdot\frac{1}{x}\,dx$$

$$= \frac{3}{4}\cdot\frac{x^2}{2} + \frac{1}{8}\ln|x| + C$$

$$= \frac{3}{8}x^2 + \frac{1}{8}\ln|x| + C$$

6. A $\dfrac{2x^3+1}{4x^2} + C$

$\boxed{\textbf{B} \quad \dfrac{3}{8}x^2 + \dfrac{1}{8}\ln|x| + C}$

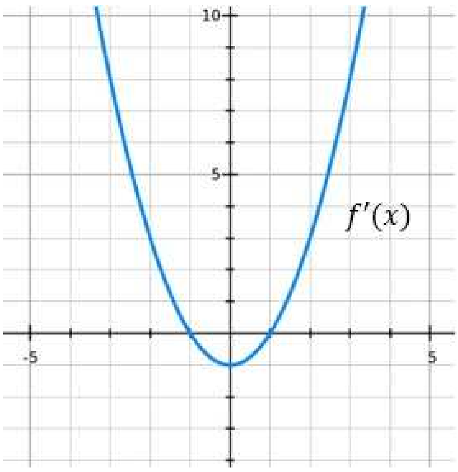
C $\dfrac{3x+1}{8x} + C$

D $\dfrac{2x^3+x}{4x^2} + C$

E $\dfrac{3}{8}x^2 + \ln|8x| + C$

F $\dfrac{3}{2}x + C$

| *8 pt* | Given the graph of $f'(x)$ below, choose the correct statement regarding $f(x)$.



Increasing: $(-\infty,-1) \cup (1,\infty)$
Decreasing: $(-1,1)$

max min
$+$ $-$ $+$ f'
-1 1
critical #s

Concave Up: $(0,\infty)$
Concave Down: $(-\infty,0)$
$+$ $-$ f''
0
Inflection Pt

7. ~~A~~ $f(x)$ is concave up on $(-\infty,\infty)$.

   ~~B~~ $f(x)$ is decreasing on $(-\infty,0)$.

   C $f(x)$ has a relative maximum at $x=-1$.

   ~~D~~ $f(x)$ does not have an inflection point.

   ~~E~~ $f(x)$ has only one critical number.

   ~~F~~ $f(x)$ has a relative minimum at $x=0$.

| *8 pt* | Evaluate

$$\int (\cos x - \sin x + \csc x \cot x)\, dx.$$

8. ~~A~~ $\sin x - \cos x + \csc x + C$

   ~~B~~ $\sin x + \cos x - \cot x + C$

   ~~C~~ $-\sin x - \cos x + \cot x + C$

   D $\sin x + \cos x - \csc x + C$

   ~~E~~ $-\sin x + \cos x + \cot x + C$

   ~~F~~ $-\sin x - \cos x - \csc x + C$

$\int \cos x\, dx = \sin x$

$\int -\sin x\, dx = \cos x$

$\int \csc x \cot x = -\csc x$

---

8 pt Given $y'' = 6e^x + 6$ with $y'(0) = 4$ and $y(2) = 6e^2$. Find $y(3)$.

9. A $6e^3 - 8$

B $6e^3 - 6$

C $6e^3 + 10$

D $6e^3 + 18$

E $6e^3 - 12$

$\boxed{\textbf{F} \quad 6e^3 + 13}$

$\int y''\,dy = \int (6e^x + 6)\,dx$

$y' = 6e^x + 6x + C$

$4 = y'(0) = 6e^0 + 6 \cdot 0 + C$

$4 = 6 + C$

$-2 = C$

$y' = 6e^x + 6x - 2$

$\int y'\,dy = \int (6e^x + 6x - 2)\,dx$

$y = 6e^x + \frac{6x^2}{2} - 2x + C$

$y = 6e^x + 3x^2 - 2x + C$

$\cancel{6e^2} = y(2) = \cancel{6e^2} + 3(2)^2 - 2(2) + C$

$0 = 12 - 4 + C$

$C = -8$

$y = 6e^x + 3x^2 - 2x - 8$

$y(3) = 6e^3 + 3(3)^2 - 2(3) - 8 = 6e^3 + 13$

---

8 pt You have 80 feet of fence to create a rectangular dog run, which will be bounded on one side by the wall of your house. What is the area of the largest dog run that you can create?

10. $\boxed{\textbf{A} \quad 800 \text{ ft}^2}$

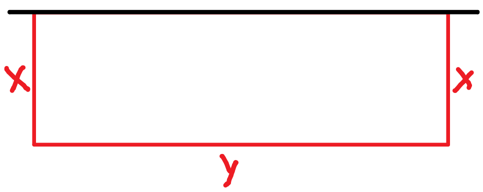
B 842 ft^2

C 748 ft^2

D 938 ft^2

E 964 ft^2

F 780 ft^2



$80 = P = 2x + y$

$y = 80 - 2x$

$A = xy$

$= x(80 - 2x)$

$= 80x - 2x^2$

$A' = 80 - 4x = 0$

$80 = 4x$

$x = 20$

$y = 80 - 2(20)$

$= 80 - 40$

$= 40$

$A = 20 \times 40$

$= 800$

---

8 pt A company's marketing department has determined that if their product is sold at the price of p dollars per unit, they can sell $q = 1200 - 100p$ units. Each unit costs 6 dollars to make. What price, p, should the company charge to maximize their profit?

11. A \$18

B \$15

C \$10

D \$12

$\boxed{\textbf{E} \quad \$9}$

F \$8

$P = R - C$

$R = pq$

$C = 6$

$P = pq - 6q$

$= q(p - 6)$

$= (1200 - 100p)(p - 6)$

$= -100(p - 12)(p - 6)$

$= -100(p^2 - 18p + 72)$

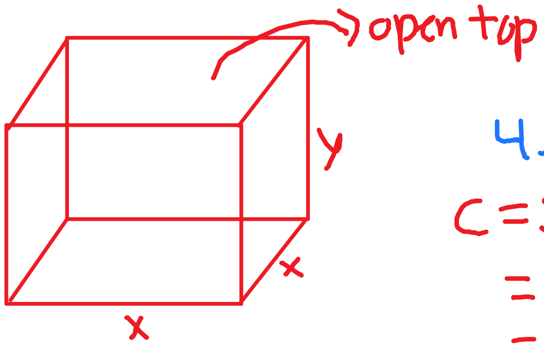
$P' = -100(2p - 18) = 0$

$2p - 18 = 0$

$p = 9$

Matt is building an open top wooden box for his cat Rupert, who likes to play with boxes. Matt is using wood on all four sides and the bottom, and is covering the wood on the bottom with carpet.

Matt would like the bottom to have a square base. The cost of wood is \$2 per ft^2 and the cost of the carpet is \$1 per ft^2. Rupert requires 48 ft^3 to have a satisfactory play in the box. What is the minimum total cost of this box?



\$2 ← Wood for sides & bottom
\$1 ← Carpet for bottom

8 pt

12. **A** \$156

B \$164

C \$118

D \$176

E \$132

$\boxed{\textbf{F} \quad \$144}$

$$48 = V = x^2 y \to y = \frac{48}{x^2}$$

$$C = \$2(x^2 + 4xy) + \$1(x^2)$$
$$= 2x^2 + 8xy + x^2$$
$$= 3x^2 + 8xy$$

$$C = 3x^2 + 8x\left(\frac{48}{x^2}\right)$$
$$= 3x^2 + \frac{384}{x}$$
$$= 3x^2 + 384x^{-1}$$
$$C' = 6x - 384x^{-2} = 0$$
$$6x - \frac{384}{x^2} = 0$$
$$\frac{6x}{1} = \frac{384}{x^2}$$
$$6x^3 = 384$$
$$x^3 = 64$$
$$x = 4$$

$$C(4) = 3(4)^2 + \frac{384}{4} = 144$$

