

When given the graph f' (Lesson 21)

- ① Critical # (s) where the graph touches/crosses the x -axis
- ② Increasing Interval(s) where the graph is above the x -axis
- ③ Decreasing Interval(s) where the graph is below the x -axis
- ④ Relative Max } Draw a # line with ② & ③ and use 1st
⑤ Relative Min } Derivative Test
- ⑥ Concave Up where the slope of f' is positive
- ⑦ Concave Down where the slope of f'' is negative
- ⑧ Inflection Pt(s) Draw a # line with ⑥ and ⑦ and check for changes in sign.

Curve Sketching

- ① Find domain of $f(x)$. i.e. When does $f(x)$ DNE?
- ② Find x -intercept(s). Set $y=0$. Solve for x .
- ③ Find y -intercept(s). Set $x=0$. Solve for y .
- ④ End Behavior Use the general rule
 - a) $\lim_{x \rightarrow \infty} f(x)$
 - b) $\lim_{x \rightarrow -\infty} f(x)$
- ⑤ Asymptotes
 - a) Vertical: ① Check that $f(x)$ is simplify
② Set denominator to 0. Solve.
 - b) Horizontal: Use ④. If a) or b) equals a #, then $y = #$
If $\lim_{x \rightarrow \pm\infty} f(x) \neq L$ a constant then there is no HA.
 - c) Slant Asymptote: ① Check that difference b/w the exponents of the leading coefficient of num. or deno. terms is equal to 1

② If so, use Synthetic Division or Long Division.

⑥ Critical # (s) : $f'(x) = 0$ or DNE (but $f(x)$ exists)

⑦ Increasing / Decreasing Intervals Use ⑥, and create # line.

⑧ Relative Extrema(s): Use ⑦ and 1st Derivative Test.

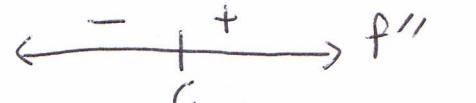
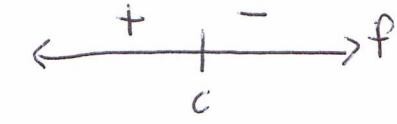
⑨ Concavity: ① Find $f''(x) = 0$ or DNE (when $f(x)$ exists)

② Create # line

③ $f''(x) > 0 \Rightarrow$ Up

$f''(x) < 0 \Rightarrow$ Down

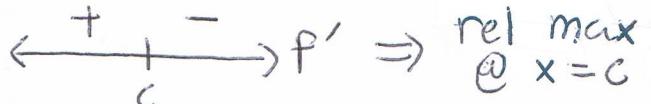
⑩ Inflection Point(s): Use ⑨ and check for change of sign.

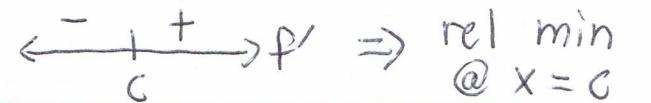
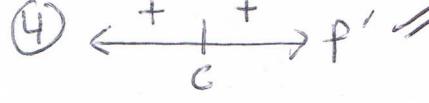
i.e.  or 

⑪ Graph

Gaps: Graph ② + ③ + ⑤ first, then use everything else.

Recall First Derivative Test: c is a critical #

①  \Rightarrow rel max @ $x=c$ ③  \Rightarrow neither

②  \Rightarrow rel min @ $x=c$ ④ 

Second Derivative Test: c is a critical #

① $f''(c) > 0 \Rightarrow$ rel min ② $f''(c) < 0 \Rightarrow$ rel max

If $f''(c) = 0$, apply the First Derivative Test.

- Absolute Extremas: ① Find all critical #s $[f'(x) = 0]$
 ② Plug ① and endpts into $f(x)$
 ③ Compare the function values and determine abs extrema
 i.e. Biggest $f(x) \Rightarrow$ abs max Smallest $f(x) \Rightarrow$ abs min
 Note if you only have 1 value from ① and no endpts,
 you must use 1st derivative test.

Product Rule: $f(x) = u(x)v(x)$

$$f'(x) = u'(x)v(x) + v'(x)u(x)$$

Quotient Rule: $f(x) = \frac{u(x)}{v(x)}$ $f'(x) = \frac{u'(x)v(x) - v'(x)u(x)}{v^2(x)}$

Chain Rule: $f(x) = a(b(x))$

$$f'(x) = a'(b(x))b'(x)$$

Some Useful Formulas

Right Triangle

Pythagorean Theorem:

$$a^2 + b^2 = c^2$$

Triangle

$$A = \frac{1}{2}bh$$

$$P = a + b + c$$

Equilateral Triangle

$$h = \frac{\sqrt{3}}{2}s$$

$$A = \frac{\sqrt{3}}{4}s^2$$

Rectangle

$$A = lw$$

$$P = 2l + 2w$$

Trapezoid

$$A = \frac{1}{2}(a + b)h$$

Circle

$$A = \pi r^2$$

$$C = 2\pi r$$

Circular Sector

$$A = \frac{1}{2}r^2\theta$$

Circular Ring

$$A = \pi(R^2 - r^2)$$

Rectangular Box

$$V = lwh$$

$$S = 2(hl + lw + hw)$$

Sphere

$$V = \frac{4}{3}\pi r^3$$

$$S = 4\pi r^2$$

Right Circular Cylinder

$$V = \pi r^2 h$$

Right Circular Cone

$$V = \frac{1}{3}\pi r^2 h$$

Recipe for Solving an Optimization Problem

Step 1: Identify what quantity you are trying to optimize.

Step 2: Draw a picture (if applicable), corresponding to the problem, and label it with your variables.

Step 3: Express the variable to be optimized as a function of the variables you used in Step 2.

Step 4: Find relations among the variables from Step 2 and express the variable to be optimized as a function of just one of the variables from Step 2.

Step 5: Identify the domain for the function you found in Step 4.

Step 6: Find the absolute maximum of the variable to be optimized on this domain.
or min

Step 7: Reread the question and be sure you have answered exactly what was asked.

This space is left for you to take your own notes.

→ Get ③ into one variable.

Take derivative, then set = 0.

Using critical pts and/or end pts, determine

absolute max/min.

Differentiation Rules

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(nx) = k$$

$$\frac{d}{dx}(nf(x)) = kf'(x)$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}, x > 0$$

Integration Rule

$$\int 0 dx = c$$

$$\int k dx = kx + C$$

$$\int kf'(x) dx = kf(x) + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \text{ when } n \neq -1$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

- For Exam 3, always + C for integration

- Always take derivative of your answer when integrating.
Especially when you have trig functions!!!

IVP: Integrate

Plug in condition

Repeat until you get $y = \dots$