When given the graph f'

(Lesson 81)

① Critical #(s) where the graph touches/crosses the x-axis

② Increasing Interval(s) where the graph is above the x-axis

③ Decreasing Interval(s) where the graph is below the x-axis

④ Relative Max } Draw a # line with ② & ③ and use 1st Derivative Test

⑤ Relative Min

⑥ Concave Up where the slope of f' is positive

⑦ Concave Down where the slope of f'' is negative

⑧ Inflection Pt(s) Draw a # line with ⑥ and ⑦ and check for changes in sign.

Curve Sketching

① Find domain of f(x). i.e. When does f(x) DNE?

② Find x-intercept(s). Set y=0. Solve for x.

③ Find y-intercept(s). Set x=0. Solve for y.

④ End Behavior use the general rule

ⓐ $\lim_{x\to\infty} f(x)$ ⓑ $\lim_{x\to-\infty} f(x)$

⑤ Asymptotes

ⓐ Vertical: ① Check that f(x) is simplify

② Set denominator to 0. Solve.

ⓑ Horizontal: Use ④. If ⓐ or ⓑ equals a #, then y=#

If $\lim_{x\to\pm\infty} f(x) \neq L$ a constant then there is no HA.

ⓒ Slant Asymptote: ① Check that difference b/w the exponents of the leading coefficient of num. or deno. terms is equal to 1

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② If so, use Synthetic Division or Long Division.

⑥ Critical #(s): $f'(x) = 0$ or DNE (but $f(x)$ exists)

⑦ Increasing / Decreasing Intervals: Use ⑥, and create # line.

⑧ Relative Extrema(s): Use ⑦ and 1st Derivative Test.

⑨ Concavity: ① Find $f''(x) = 0$ or DNE (when $f(x)$ exists)

② Create # line

③ $f''(x) > 0 \Rightarrow$ Up

$f''(x) < 0 \Rightarrow$ Down

⑩ Inflection Point(s): Use ⑨ and check for change of sign.

i.e. $\overset{-}{\longleftarrow}\!\!\underset{c}{|}\!\!\overset{+}{\longrightarrow}$ f'' or $\overset{+}{\longleftarrow}\!\!\underset{c}{|}\!\!\overset{-}{\longrightarrow}$ f''

⑪ Graph

~~Tips~~: Graph ② + ③ + ⑤ first, then use everything else.

Recall First Derivative Test: c is a critical #

① $\overset{+}{\longleftarrow}\!\!\underset{c}{|}\!\!\overset{-}{\longrightarrow}$ $f' \Rightarrow$ rel max @ $x = c$

② $\overset{-}{\longleftarrow}\!\!\underset{c}{|}\!\!\overset{+}{\longrightarrow}$ $f' \Rightarrow$ rel min @ $x = c$

③ $\overset{-}{\longleftarrow}\!\!\underset{c}{|}\!\!\overset{-}{\longrightarrow}$ $f' \Rightarrow$ neither

④ $\overset{+}{\longleftarrow}\!\!\underset{c}{|}\!\!\overset{+}{\longrightarrow}$ $f' \Rightarrow$ neither

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Second Derivative Test: c is a critical #

① $f''(c) > 0 \Rightarrow$ rel min ② $f''(c) < 0 \Rightarrow$ rel max

If $f''(c) = 0$, apply the First Derivative Test.

Absolute Extremas: ① Find all critical #s [$f'(x)=0$]
② Plug ① and endpts into $f(x)$
③ Compare the function values and determine abs extrema
i.e. Biggest $f(x) \Rightarrow$ abs max Smallest $f(x) \Rightarrow$ abs min
Note if you only have 1 value from ① and no endpts, you must use 1st derivative test.

Product Rule: $f(x)=u(x)v(x)$
$f'(x)=u'(x)v(x)+v'(x)u(x)$

Quotient Rule: $f(x)=\frac{u(x)}{v(x)}$ $f'(x)=\frac{u'(x)v(x)-v'(x)u(x)}{v^2(x)}$

Chain Rule: $f(x)=a(b(x))$
$f'(x)=a'(b(x))b'(x)$

Some Useful Formulas

Right Triangle
Pythagorean Theorem:
$a^2+b^2=c^2$

Triangle
$A=\frac{1}{2}bh$
$P=a+b+c$

Equilateral Triangle
$h=\frac{\sqrt{3}}{2}s$ $A=\frac{\sqrt{3}}{4}s^2$

Rectangle
$A=lw$
$P=2l+2w$

Trapezoid
$A=\frac{1}{2}(a+b)h$

Circle
$A=\pi r^2$ $C=2\pi r$

Circular Sector
$A=\frac{1}{2}r^2\theta$ $s=r\theta$

Circular Ring
$A=\pi(R^2-r^2)$

Rectangular Box
$V=lwh$
$S=2(hl+lw+hw)$

Sphere
$V=\frac{4}{3}\pi r^3$ $S=4\pi r^2$

Right Circular Cylinder
$V=\pi r^2h$ $S=2\pi rh$

Right Circular Cone
$V=\frac{1}{3}\pi r^2h$

Recipe for Solving an Optimization Problem

Step 1: Identify what quantity you are trying to optimize.

Step 2: Draw a picture (if applicable), corresponding to the problem, and label it with your variables.

Step 3: Express the variable to be optimized as a function of the variables you used in Step 2.

Step 4: Find relations among the variables from Step 2 and express the variable to be optimized a function of just one of the variables from Step 2.

Step 5: Identify the domain for the function you found in Step 4.

Step 6: Find the absolute maximum of the variable to be optimized on this domain.
or min

Step 7: Reread the question and be sure you have answered exactly what was asked.

This space is left for you to take your own notes.

→ Get 3 into one variable.

Take derivative, then set $= 0$.

Using critical pts and/or end pts, determine

absolute max/min.

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Differentiation Rules	Integration Rule
$\frac{d}{dx}(c)=0$	$\int 0\,dx=C$
$\frac{d}{dx}(kx)=k$	$\int k\,dx=kx+C$
$\frac{d}{dx}(kf(x))=kf'(x)$	$\int kf'(x)\,dx=kf(x)+C$
$\frac{d}{dx}(x^n)=nx^{n-1}$	$\int x^n\,dx=\frac{x^{n+1}}{n+1}+C$ when $n\neq -1$
$\frac{d}{dx}(\sin x)=\cos x$	$\int \cos x\,dx=\sin x+C$
$\frac{d}{dx}(\cos x)=-\sin x$	$\int \sin x\,dx=-\cos x+C$
$\frac{d}{dx}(\tan x)=\sec^2 x$	$\int \sec^2 x\,dx=\tan x+C$
$\frac{d}{dx}(\cot x)=-\csc^2 x$	$\int \csc^2 x\,dx=-\cot x+C$
$\frac{d}{dx}(\sec x)=\sec x\tan x$	$\int \sec x\tan x\,dx=\sec x+C$
$\frac{d}{dx}(\csc x)=-\csc x\cot x$	$\int \csc x\cot x\,dx=-\csc x+C$
$\frac{d}{dx}(e^x)=e^x$	$\int e^x\,dx=e^x+C$
$\frac{d}{dx}(\ln x)=\frac{1}{x}$, $x>0$	$\int \frac{1}{x}\,dx=\ln\lvert x\rvert+C$

- For Exam 3, always $+C$ for integration
- Always take derivative of your answer when integrating. Especially when you have trig functions!!!

IVP: Integrate
Plug in condition
Repeat until you get $y=\ldots$

E3P5