Lesson 13: Higher Order Derivatives

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The derivative of a function, $f(x)$, is also called first derivative.

$$y', \quad f'(x), \quad \frac{dy}{dx}, \quad \frac{d}{dx}[f(x)]$$

If we take the derivative of the first derivative of $y=f(x)$, then we get second derivative

$$y'', \quad f''(x), \quad \frac{d^2y}{dx^2}, \quad \frac{d^2}{dx^2}[f(x)]$$

Take the derivative n times, we get the nth derivative.

$$y^{(n)}, \quad f^{(n)}(x), \quad \frac{d^ny}{dx^n}, \quad \frac{d^n}{dx^n}[f(x)]$$

Ex 1: Find the first three derivatives of

$$R(x) = 3x^2 + 8x^{1/2} + e^x$$

① $R'(x) = 6x + 8\left(\frac{1}{2}\right)x^{-1/2} + e^x$

$= 6x + 4x^{-1/2} + e^x$

② $R''(x) = 6 + 4\left(-\frac{1}{2}\right)x^{-3/2} + e^x$

$= 6 - 2x^{-3/2} + e^x$

③ $R'''(x) = R^{(3)}(x) = -2\left(-\frac{3}{2}\right)x^{-5/2} + e^x$

$= 3x^{-5/2} + e^x = \frac{3}{x^{5/2}} + e^x$

Ex 2: Find the first five derivatives of $y = \sin x$.

Recall $\frac{d}{dx}[\sin x] = \cos x$ & $\frac{d}{dx}[\cos x] = -\sin x$

① $y' = \cos x$

② $y'' = -\sin x$

③ $y''' = y^{(3)} = -\frac{d}{dx}[\sin x] = -\cos x$

④ $y^{(4)} = -\frac{d}{dx}[\cos x] = -(-\sin x) = \sin x$

⑤ $y^{(5)} = \cos x$

Note if $y = \cos x$, a similar pattern occurs.

Ex 3: Given $f(x) = xe^x$. Find $f''(x)$ and $f^{(3)}(x)$

① Find f'.

$u(x) = x \quad v(x) = e^x$

$u'(x) = 1 \quad v'(x) = e^x$

By product rule,

$$f'(x) = u'(x)v(x) + u(x)v'(x)$$
$$= 1 \cdot e^x + xe^x$$
$$= (1+x)e^x$$

② Find f''. We do this by taking derivative of f'!

$u(x) = 1+x \quad v(x) = e^x$

$u'(x) = 1 \quad v'(x) = e^x$

By Product Rule,

$$f''(x) = u'(x)v(x) + u(x)v'(x)$$

$$= 1 \cdot e^x + (1+x)e^x$$
$$= (1+1+x)e^x$$
$$= (2+x)e^x$$

③ Find $f^{(3)}$:

$u(x) = 2+x \quad v(x) = e^x$

$u'(x) = 1 \quad v'(x) = e^x$

By Product Rule,

$$f^{(3)}(x) = u'(x)v(x) + u(x)v'(x)$$
$$= 1 \cdot e^x + (2+x)e^x$$
$$= (1+2+x)e^x$$
$$= (3+x)e^x$$

Position & Velocity & Acceleration Functions

Recall that $\boxed{v(t) = s'(t)}$

Acceleration Function [$a(t)$] tells us how fast the velocity changes

Hence

$$\boxed{a(t) = v'(t) = (s'(t))' = s''(t)}$$

Ex 4: The position function of a particle is

$$s(t) = \frac{1}{12}t^4 - \frac{4}{3}t^3 + 8t^2 - 64t$$

What is the acceleration of the particle @ $t = 2$?

i.e. Find $a(2) = s''(2)$

$$s'(t) = \frac{4}{12}t^3 - \frac{4}{3}(3)t^2 + 8(2)t - 64$$

$$= \frac{1}{3}t^3 - 4t^2 + 16t - 64$$

Recall I want $s''(2)$!

$$s''(t) = \frac{3}{3}t^2 - 4(2)t + 16 = t^2 - 8t + 16$$

$$s''(2) = 2^2 - \cancel{8(2)} + \cancel{16}$$

$$= \boxed{4}$$