Lesson 17: Relative Extrema & Critical Numbers

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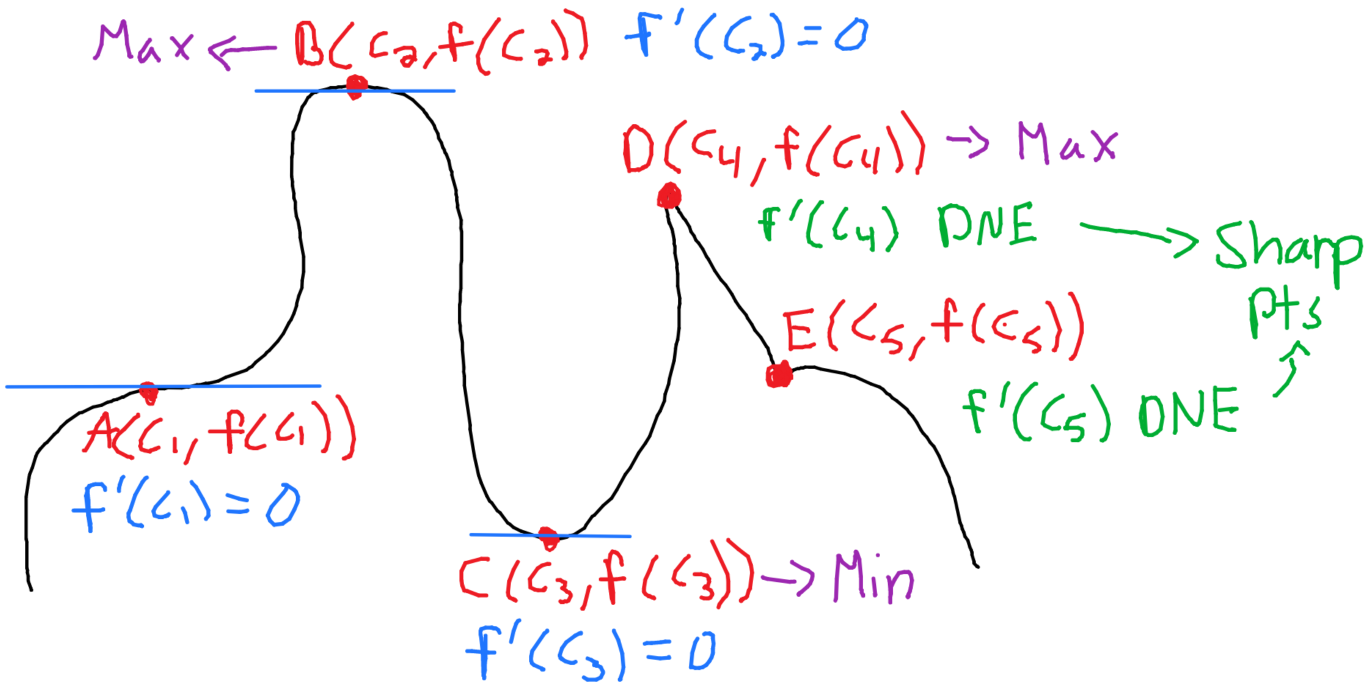
Definition: ⓐ If $\boxed{f(c) \geq f(x)}$ for all x in an open interval I containing c, then f(c) is a relative maximum.

ⓑ If $\boxed{f(c) \leq f(x)}$ for all x in an open interval I containing c, then f(c) is a relative minimum.



Overall, all relative extremas occur at points where the derivative is zero or DNE

Question: Is the reverse true?
i.e. If the derivative is zero or DNE, do we have a relative extrema?

No. Refer back to Pts A and E from the graph

Definition: Let c be a # in the domain of f. If $\boxed{f'(x)=0}$ or $\boxed{f'(x) \text{ DNE}}$ @ $x=c$, then c is a critical number.

Ex 1: Find the critical #s of $y=x^4-2x^3$

i.e. Solve $y'(x)=0$ for x.

$$y'=4x^3-6x^2=0$$
$$2x^2(2x-3)=0$$
$$2x^2=0 \quad \Big| \quad 2x-3=0$$
$$x=0 \quad \Big| \quad x=3/2$$

Since the domain of y is $(-\infty,\infty)$ then $x=0$, $x=-\frac{3}{2}$ are critical pts.

Ex 2: Find the critical #s of $y=5x^{4/5}$

i.e. Solve $y'(x)=0$ for x.

$$y'=\cancel{5}\cdot\frac{4}{\cancel{5}}x^{-1/5}=0$$
$$\frac{4}{x^{1/5}}=0$$
$$4=0 \Rightarrow y'(x)\neq 0$$

Since the domain of y is $[0,\infty)$, $x=0$ is the critical point.

When does y' DNE?

$y' = \frac{4}{x^{1/5}}$. So $x=0$, y' DNE

Ex 3: Find the critical #s of $y = 3x^4 e^x$

i.e. Solve $y'(x) = 0$ for x.

$u(x) = 3x^4 \qquad v(x) = e^x$

$u'(x) = 12x^3 \qquad v'(x) = e^x$

$y' = 12x^3 e^x + 3x^4 e^x = 0$

$3x^3 e^x (4+x) = 0$

$3x^3 = 0$	$e^x = 0$	$4+x=0$
$x=0$	Never	$x=-4$

Since the domain of y is $(-\infty, \infty)$, then $x=0$, $x=-4$ are critical pts.

Ex 4: Is $\pi/8$ a critical number for $y = 2\sin(2x) - 2\sqrt{2}x$?

$\boxed{\text{Yes}}$

i.e. Check $y'(\pi/8) = 0$

$y' = 2\cos(2x) \cdot 2 - 2\sqrt{2}$

$= 4\cos(2x) - 2\sqrt{2}$

$y'\left(\frac{\pi}{8}\right) = 4\cos\left(\frac{2\pi}{8}\right) - 2\sqrt{2}$

$= 4\cos\left(\frac{\pi}{4}\right) - 2\sqrt{2}$

$= \overset{2}{\cancel{4}}\left(\sqrt{2}/\cancel{2}\right) - 2\sqrt{2} = 2\sqrt{2} - 2\sqrt{2} = 0$