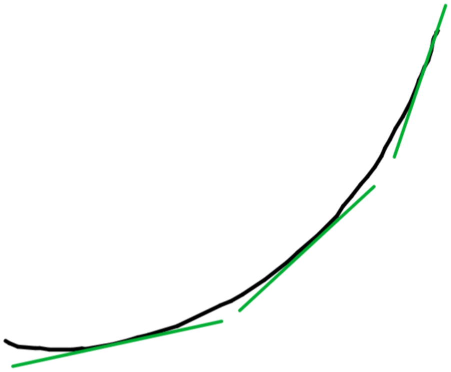
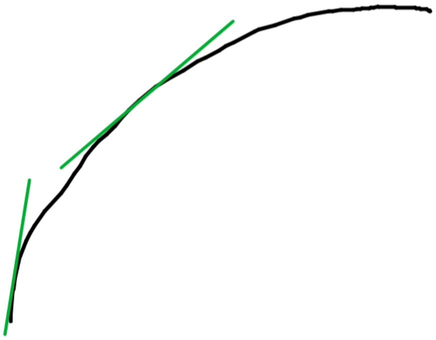
Lesson 18: Increasing and Decreasing Functions & the First Derivative Test

Lesson 18: Increasing and Decreasing Functions and the First Derivative Test

A function is increasing if the function value gets bigger and bigger.

A function is decreasing if the function value gets smaller and smaller.





Notice that slope of the tangent lines when f(x)

increasing ⇒ positive

decreasing ⇒ negative

Theorem: Let f(x) be a continuous and differentiable function on an open interval, I.

- If $\boxed{f'(x)>0}$ for all x in I, then f(x) is increasing on I.

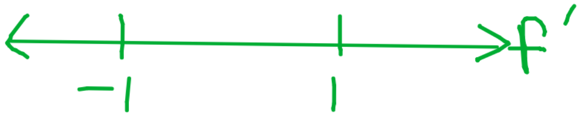
- If $\boxed{f'(x)<0}$ for all x in I, then f(x) is decreasing on I.

Ex 1: Given $f(x)=x^3-3x$. Find where f is inc/dec

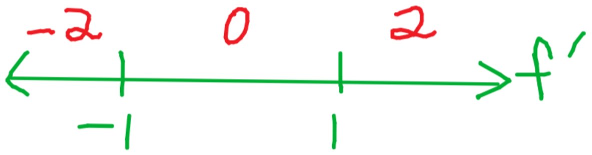
Step 1: Find when $f'(x)=0$

$$f'(x)=3x^2-3=0$$
$$3(x^2-1)=0$$
$$3(x-1)(x+1)=0$$
$$x-1=0 \quad\Big|\quad x+1=0$$
$$x=1 \quad\Big|\quad x=-1$$

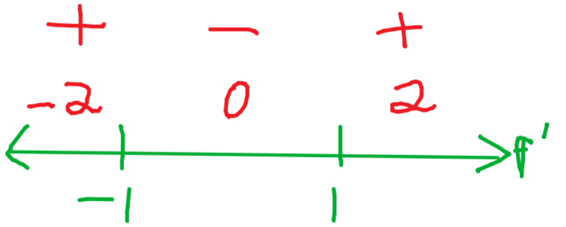
Step 2: Draw a # line with the pts from ①



Step 3: Determine test pts using ②.



Step 4: Plug those values into f' to determine whether it's positive or negative

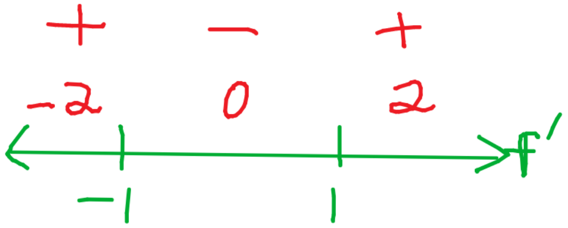


$$f'(x)=3(x-1)(x+1)$$
$$f'(-2)=3(-2-1)(-2+1)$$
$$+\cdot-\cdot-=+$$
$$f'(0)=3(0-1)(0+1)$$
$$+\cdot-\cdot+=-$$

$$f'(2) = 3(2-1)(2+1)$$

$$+ \cdot + \cdot + = +$$

Step 5: Use definition of increasing/decreasing



Increasing: $(-\infty,-1)\cup(1,\infty)$

Decreasing: $(-1,1)$

Ex2: Given $f(x)=2x^2e^{4x+1}$. Find where f is inc/dec

Step 1: Find when $f'(x)=0$

$u(x)=2x^2 \qquad v(x)=e^{4x+1}$

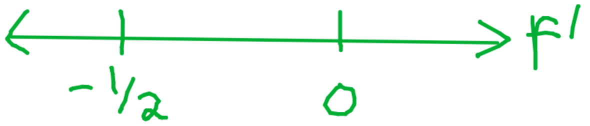
$u'(x)=4x \qquad v'(x)=4e^{4x+1}$

$$f'(x)=2x^2(4e^{4x+1})+4xe^{4x+1}=0$$

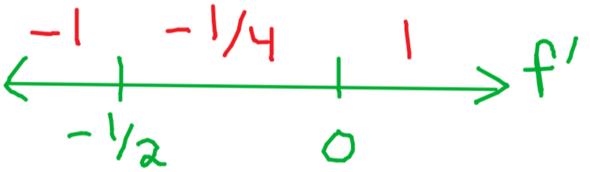
$$4xe^{4x+1}[2x+1]=0$$

$4x=0$	$e^{4x+1}=0$	$2x+1=0$
$x=0$	Never	$x=-1/2$

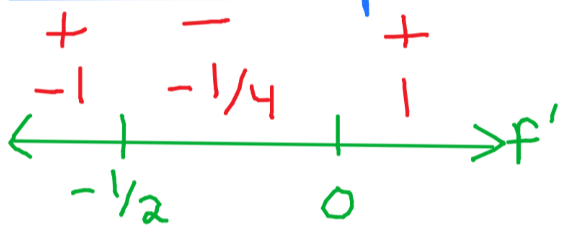
Step 2: Draw a # line with the pts from ①



Step 3: Determine test pts using ②.



Step 4: Plug those values into f' to determine whether it's positive or negative



$f'(x) = 4xe^{4x+1}[2x+1]$

$f'(-1) = 4(-1)e^{-4+1}(-2+1)$

$- \cdot + \cdot - = +$

$f'(-1/4) = 4(-1/4)e^{4(-1/4+1)}(2(-\frac{1}{4})+1)$

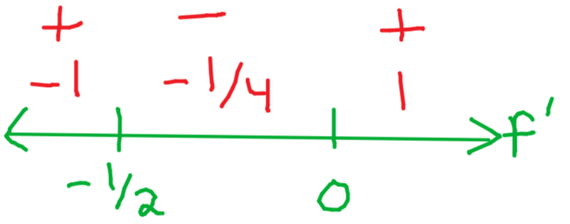
$- \cdot + \cdot + = -$

$f'(1) = 4e^{4+1}(2+1)$

$+ \cdot + \cdot + = +$

$\boxed{\text{Note } e^x > 0 \text{ for all } x}$

Step 5: Use definition of increasing/decreasing



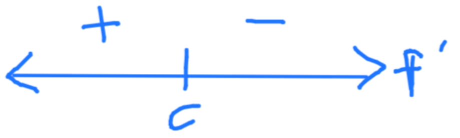
Increasing: $(-\infty, -\frac{1}{2}) \cup (0, \infty)$

Decreasing: $(-\frac{1}{2}, 0)$

The First Derivative Test

Let c be a critical # of f(x) that is continuous on an open interval, I, containing c.

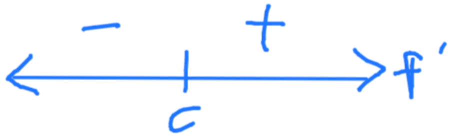
①



ex.

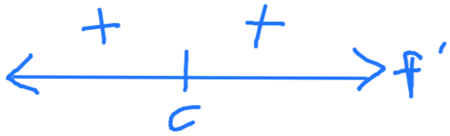
⇒ relative max @ x=c

②



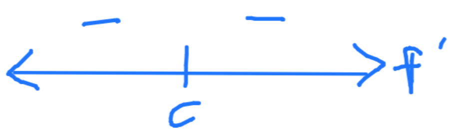
ex.  ⇒ relative min @ $x=c$

③



ex.  ⇒ neither

④



ex.  ⇒ neither

Ex 3: Given $f(x)=2x^4-2x^3$.

ⓐ Find where f is inc/dec

Step 1: Find when $f'(x)=0$

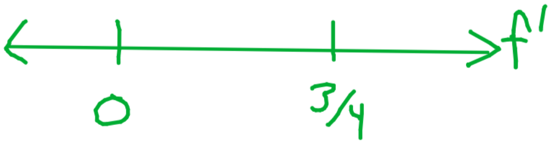
$$f'(x)=8x^3-6x^2=0$$

$$2x^2(4x-3)=0$$

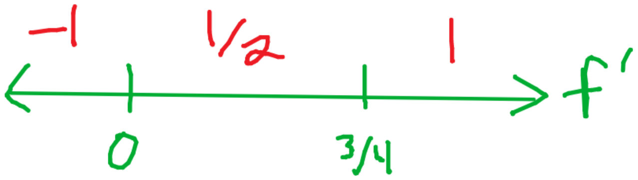
$$2x^2=0 \quad \Big| \quad 4x-3=0$$

$$x=0 \quad \Big| \quad x=3/4$$

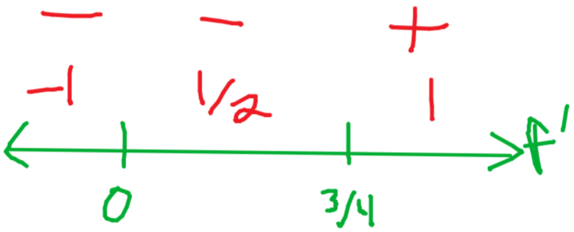
Step 2: Draw a # line with the pts from ①



Step 3: Determine test pts using ②.



Step 4: Plug those values into f' to determine whether it's positive or negative



$f'(x)=2x^2(4x-3)$

$f'(-1)=2(-1)^2(4(-1)-3)$

$=+\cdot-=-$

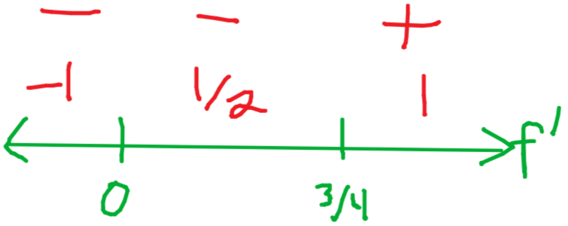
$f'(\frac{1}{2})=2(\frac{1}{2})^2(4(\frac{1}{2})-3)$

$=+\cdot-=-$

$f'(1)=2(1)^2(4-3)$

$+\cdot+=+$

Step 5: Use definition of increasing/decreasing



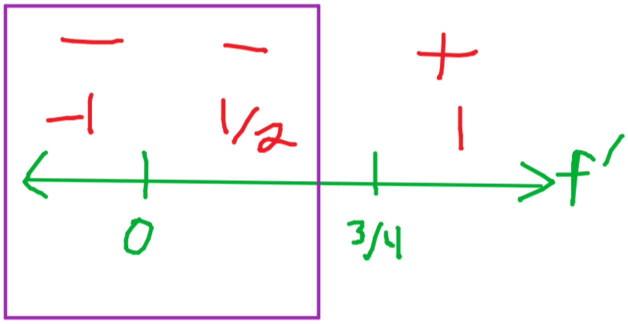
Increasing: $(3/4,\infty)$

Decreasing: $(-\infty,3/4)$

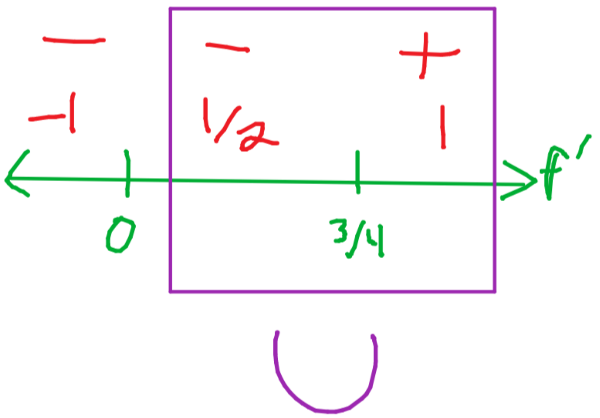
Ex 3: Given $f(x)=2x^4-2x^3$.

ⓑ Find relative extrema of f(x).

Using the # line, found in ⓐ, and the First Derivative Test.



By Case 4, $x=0$ is not relative extrema.



By Case 2, $x = 3/4$ is a relative min

Ex 4: Given $f'(x) = (3x+6)^2(x-5)$

ⓐ Find where f is inc/dec

Step 1: Find when $f'(x) = 0$

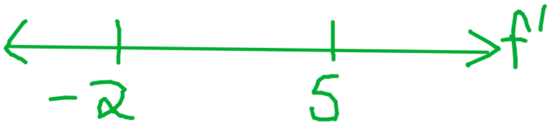
$$f'(x) = (3x+6)^2(x-5) = 0$$

$(3x+6)^2 = 0$ | $x - 5 = 0$

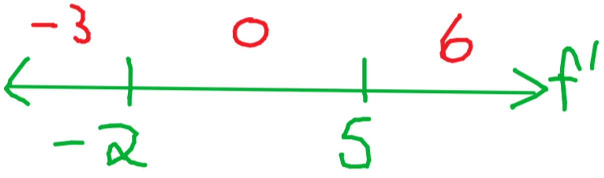
$3x + 6 = 0$ | $x = 5$

$x = -2$

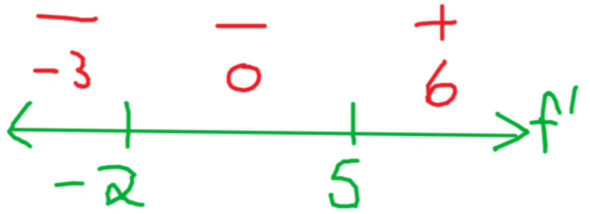
Step 2: Draw a # line with the pts from ①



Step 3: Determine test pts using ②.



Step 4: Plug those values into f' to determine whether it's positive or negative



$f'(-3)=(3(-3)+6)^2(-3-5)$

$+ \cdot - = -$

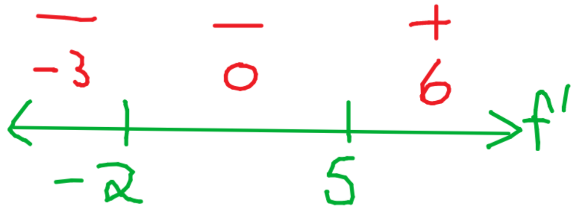
$f'(0)=(3(0)+6)^2(0-5)$

$+ \cdot - = -$

$f'(6)=(3(6)+6)^2(6-5)$

$+ \cdot + = +$

Step 5: Use definition of increasing/decreasing

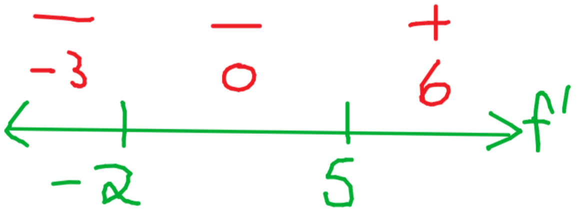


Increasing: $(5,\infty)$

Decreasing: $(-\infty,5)$

ⓑ Find relative extrema of f(x).

Using the # line, found in ⓐ, and the First Derivative Test.



By Case 4, $x=-2$ is not relative extrema.

By Case 2, $x=5$ is a relative min