Lesson 20: Absolute Extrema on an Interval

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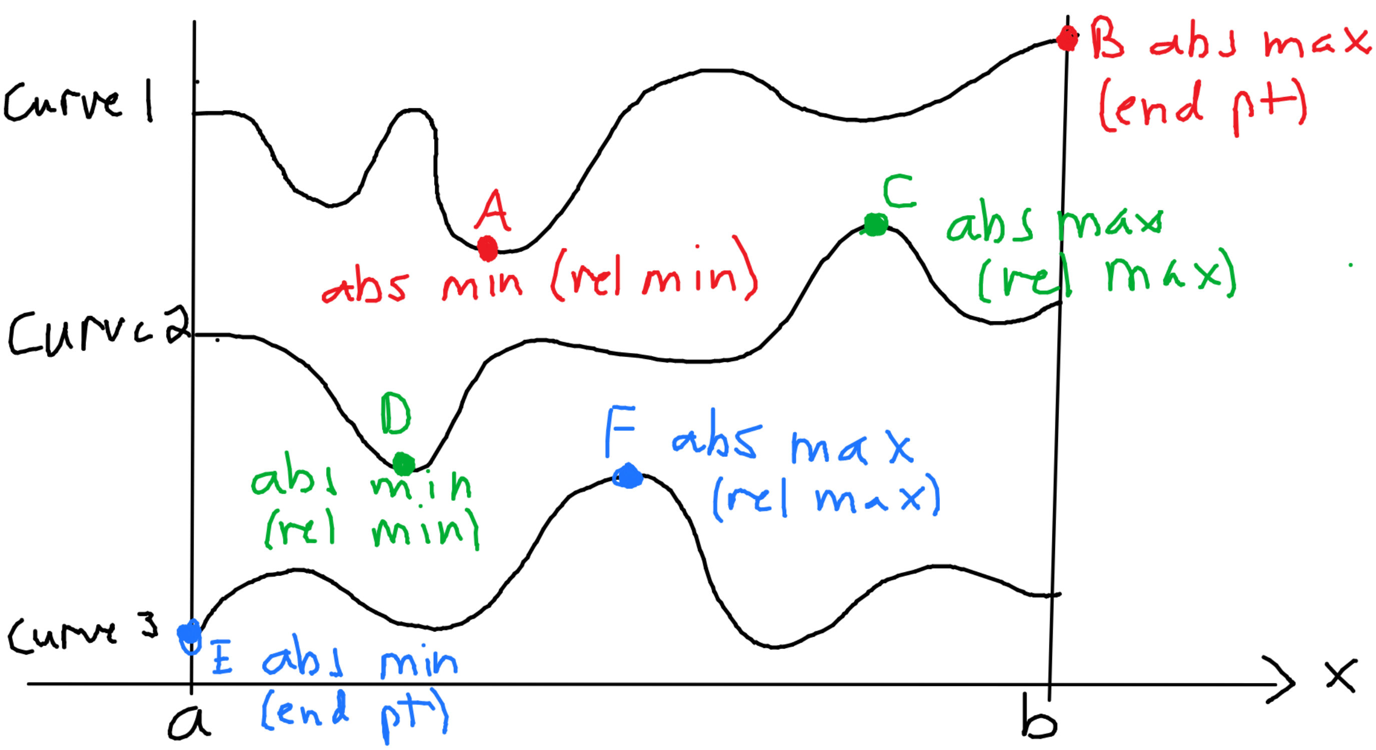
an absolute max is the largest function value on the entire interval.

an absolute min is the smallest function value on the entire interval.



Theorem: If $f(x)$ is continuous on a closed interval $[a,b]$, then $f(x)$ has both an absolute max and min on the interval.

The abs extrema only occur either @
- critical #s, or
- end pts

Note Relative $\Leftrightarrow$ Local Extremas
Absolute $\Leftrightarrow$ Global Extremas

Steps to find Absolute Extrema

(1) Find all critical #s. $[f'(x)=0]$
(2) Check if pts from (1) are in the interval given.
(3) Plug (2) and endpts into $f(x)$.
(4) Compare the function values and determine abs extrema.

[i.e. Biggest $f(x)$ value in (2) $\Rightarrow$ abs max
Smallest $f(x)$ value in (2) $\Rightarrow$ abs min]

Ex 1: Find the abs extrema of
$y = x^4 - 2x^3$ on $[-1, 1]$

Step 1: Find when $f'(x) = 0$

$y' = 4x^3 - 6x^2 = 0$

$2x^2(2x-3) = 0$

$x = 0, \frac{3}{2}$

Step 2: Are $0, \frac{3}{2}$ in $[-1,1]$?

Only 0.

Step 3: Plug ② and endpts into $f(x)$.

x	$y = x^4 - 2x^3$	Conclusion
-1	$1-2(-1) = 3$	
0	0	
1	$1-2=-1$	

Step 4: Conclusions

x	$y = x^4 - 2x^3$	Conclusion
-1	$1-2(-1) = 3$	Abs max
0	0	
1	$1-2=-1$	Abs min

Ex 2: Find the abs extrema of $y = xe^x$ on $[-2, 0]$

Step 1: Find when $f'(x) = 0$

$u(x) = x \quad v(x) = e^x$

$u'(x) = 1 \quad v'(x) = e^x$

$y' = 1 \cdot e^x + xe^x = 0$

$e^x(1 + x) = 0$

$e^x = 0$	$x = -1$
Never	

Step 2: Is -1 in $[-2, 0]$?

Yes.

Step 3: Plug ② and endpts into $f(x)$.

x	$y = xe^x$	Conclusion
-2	$-2e^{-2} = -2/e^2$	
-1	$-e^{-1} = -1/e = -\frac{e}{e^2}$	
0	0	

Step 4: Conclusions

x	$y = xe^x$	Conclusion
-2	$-2e^{-2} = -2/e^2$	Abs min
-1	$-e^{-1} = -1/e = -\frac{e}{e^2}$	
0	0	Abs max

Ex 3: Find the abs extrema of
$$y = \frac{6x^2}{x+1} \quad \text{on } (-1, 5]?$$

Step 1: Find when $f'(x) = 0$

$u(x) = 6x^2 \quad v(x) = x+1$

$u'(x) = 12x \quad v'(x) = 1$

$$y' = \frac{12x(x+1) - 1(6x^2)}{(x+1)^2} = 0$$

$$= \frac{12x^2 + 12x - 6x^2}{(x+1)^2} = 0$$

$$\frac{6x^2 + 12x}{(x+1)^2} = 0$$

$6x^2 + 12x = 0$

$6x(x+2) = 0$

$6x = 0 \quad | \quad x+2 = 0$

$x = 0 \quad | \quad x = -2$

Step 2: Are $-2, 0$ in $(-1, 5]$?

Only 0

Step 3: Plug (2) and endpts into $f(x)$.

x	$y = 6x^2/(x+1)$	Conclusion
0	$y = 6(0)^2/(0+1) = 0$	
5	$y = \frac{6(5)^2}{5+1} = 5^2 = 25$	

Step 4: Conclusions

x	$y = 6x^2/(x+1)$	Conclusion
0	$y = 6(0)^2/(0+1) = 0$	Abs min
5	$y = \frac{6(5)^2}{5+1} = 5^2 = 25$	Abs max

Ex 4: Find the abs extrema of
$y = -x^2 - 2x$ on $(-2, 0)$

Step 1: Find when $f'(x) = 0$

$y' = -2x - 2 = 0$
$-2(x+1) = 0$
$x = -1$

Step 2: Is -1 in $(-2, 0)$?

Yes

Step 3: Plug ② and endpts into $f(x)$.

x	$y = -x^2 - 2$	Conclusion
-1	$y = -1 - 2 = -3$	

Step 4: Conclusions

When you get one value as seen in Step 3, you need to use 1^{st} or 2^{nd} Derivative Test at that value.

By 2^{nd} Derivative Test

$y' = -2x - 2$
$y'' = -2$

So $y''(-1) = -2 < 0 \Rightarrow$ relative max @ $x = -1$
$\Rightarrow$ absolute max @ $x = -1$