MA 16010 LESSONS 21: GRAPHICAL INTERPRETATION OF DERIVATIVES

So far, we have learned that:

x-value	**1. Critical Point/Number:**	x-values where $f'(x) = 0$ or $f'(x)$ DNE
interval	**2. Increasing:**	$f'(x) > 0$
interval	**3. Decreasing:**	$f'(x) < 0$
x-value	**4. Relative Max:**	Create # line from (2) and (3), and then
x-value	**5. Relative Min:**	use First Derivative Test
interval	**6. Concave Up:**	$f''(x) > 0$
interval	**7. Concave Down:**	$f''(x) < 0$
x-value	**8. Inflection Point:**	Check for change of concavity from results of (6) and (7)

1. Given the graph of $f'(x)$ below, answer the following question for $f(x)$.
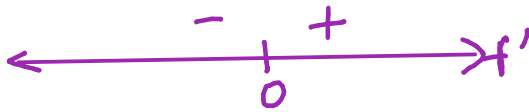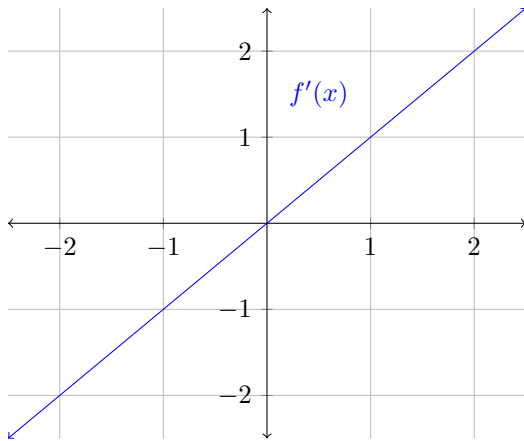
$f'(x)$

(a) **Critical Number(s):**

$$X = 0$$

(b) **Increasing Interval(s):**

$$(0, \infty)$$

(c) **Decreasing Interval(s):**

$$(-\infty, 0)$$

(d) **Relative Maximum Occurs:**

None

(e) **Relative Minimum Occurs:**

$$X = 0$$

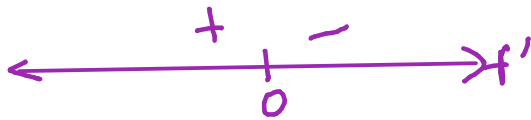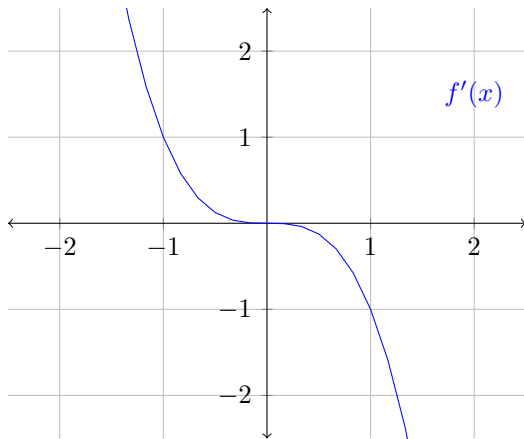(f) **Concave Up Interval(s):**

$$(-\infty, \infty)$$

(g) **Concave Down Interval(s):**

None

(h) **Inflection Point(s):**

None

$-+$
f'
0

$+$
f''

2. Given the graph of $f'(x)$ below, answer the following question for $f(x)$.

$f'(x)$

(a) **Critical Number(s):**

$$X = 0$$

(b) **Increasing Interval(s):**

$$(-\infty, 0]$$

(c) **Decreasing Interval(s):**

$$[0, \infty)$$

(d) **Relative Maximum Occurs:**

$$X = 0$$

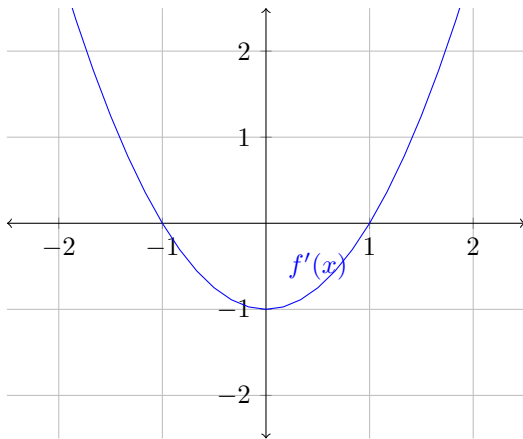(e) **Relative Minimum Occurs:**

None

(f) **Concave Up Interval(s):**

None

(g) **Concave Down Interval(s):**

$$(-\infty, \infty)$$

(h) **Inflection Point(s):**

None

$+ \quad -$

$\xrightarrow{\hspace{3cm}} f'$

0

$\xrightarrow{\hspace{3cm}} f''$

3. Given the graph of $f'(x)$ below, answer the following question for $f(x)$.

(a) **Critical Number(s):**

$$x = -1, 1$$

(b) **Increasing Interval(s):**

$$(-\infty, -1) \cup (1, \infty)$$

(c) **Decreasing Interval(s):**

$$(-1, 1)$$

(d) **Relative Maximum Occurs:**

$$x = -1$$

(e) **Relative Minimum Occurs:**

$$x = 1$$

(f) **Concave Up Interval(s):**
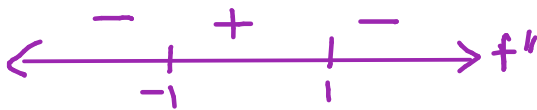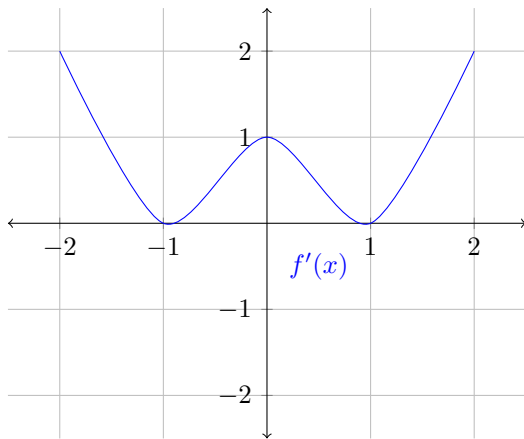
$$(0, \infty)$$

(g) **Concave Down Interval(s):**

$$(-\infty, 0)$$

(h) **Inflection Point(s):**

$$x = 0$$

4. Given the graph of $f'(x)$ below, answer the following question for $f(x)$.

$f'(x)$

(a) **Critical Number(s):**

$x = -1, 1$

(b) **Increasing Interval(s):**

$(-\infty, \infty)$

(c) **Decreasing Interval(s):**

None

(d) **Relative Maximum Occurs:**

None

(e) **Relative Minimum Occurs:**

None

(f) **Concave Up Interval(s):**

$(-1, 0) \cup (1, \infty)$

(g) **Concave Down Interval(s):**

$(-\infty, -1) \cup (0, 1)$

(h) **Inflection Point(s):**

$x = -1, 1$

f'

f''

The End.

Summary: When given the graph of f' ,

1. **Critical Point/Number:**   Where the graph touches/crosses the x-axis

2. **Increasing:**   Where the graph is above the x-axis

3. **Decreasing:**   Where the graph is below the x-axis

4. **Relative Max:**   Create # line from (2) and (3), and
5. **Relative Min:**   then use First Derivative Test

6. **Concave Up:**   Where the slope of f' is positive

7. **Concave Down:**   Where the slope of f' is negative

8. **Inflection Point:**   Create a # line with (6) and (7) and check for changes in sign