Lesson 22: Limits at Infinity Pt 1

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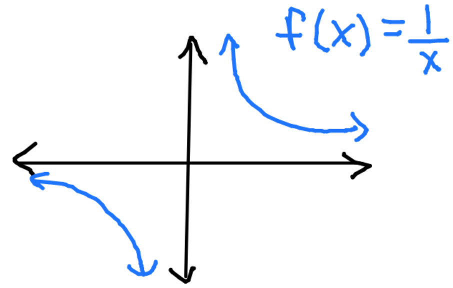
Recall

$\lim_{x \to 0^-} \frac{1}{x} = -\infty \qquad \lim_{x \to 0^+} \frac{1}{x} = \infty$



Now we want

$\lim_{x \to -\infty} \frac{1}{x} = 0 \qquad \lim_{x \to \infty} \frac{1}{x} = 0$

Purpose of revisiting this topic is to determine

1. End Behavior
2. If we have Horizontal/Slant Asymptote

Ex 1: Find the following limits

ⓐ $\lim_{x \to \infty} \frac{3}{x} = 3 \lim_{x \to \infty} \frac{1}{x} = 3 \cdot 0 = 0$

ⓑ $\lim_{x \to -\infty} \left(\frac{x}{3} + 2\right) = -\frac{\infty}{3} + 2 = -\infty + 2 = -\infty$

ⓒ $\lim_{x \to \infty} \left(\frac{x}{2} + \frac{5}{x}\right) = \frac{\infty}{2} + 5 \cdot 0 = \frac{\infty}{2} = \infty$

General Rule: The limit of a rational function $f(x) = \frac{p(x)}{q(x)}$ as $x \to \pm\infty$ is determined by the leading terms of the numerator and the denominator.

A leading term of a polynomial is the term that has the highest power of x.

Ex 2: Find $\lim_{x\to\infty} \frac{2x^2+3}{x^2-1}$

$\lim_{x\to\infty} \frac{2x^2+3}{x^2-1} = \text{"}\frac{\infty}{\infty}\text{"} \to$ No-no

By General Rule,

$$\lim_{x\to\infty} \frac{2x^2}{x^2} = \lim_{x\to\infty} 2 = \boxed{2}$$

Horizontal Asymptotes

The line $y=L$, where L is a constant is a horizontal asymptote of $f(x)$ if

$$\lim_{x\to\infty} f(x) = L \quad \boxed{\text{or}} \quad \lim_{x\to-\infty} f(x) = L$$

↓

Check both cases

Both limits don't need to match.

For example when $f(x) = e^x$

Ex 3: Let $h(x) = \frac{x-1}{x^2-1}$. Find the HA.

Method 1: Simplify $h(x)$, first.

$$h(x) = \frac{\cancel{x-1}}{(\cancel{x-1})(x+1)} = \frac{1}{x+1}$$

$$\lim_{x \to -\infty} h(x) = \lim_{x \to -\infty} \frac{1}{x+1} = 0 \rightarrow$$

$$\lim_{x \to \infty} h(x) = \lim_{x \to \infty} \frac{1}{x+1} = 0 \rightarrow$$

HA $y=0$

Method 2: Using the general rule

$$\lim_{x \to -\infty} h(x) = \lim_{x \to -\infty} \frac{x}{x^2} = \lim_{x \to -\infty} \frac{1}{x} = 0 \rightarrow$$

$$\lim_{x \to \infty} h(x) = \lim_{x \to \infty} \frac{x}{x^2} = \lim_{x \to \infty} \frac{1}{x} = 0 \rightarrow$$

HA @ $y=0$

Ex 4: Let $h(x) = \frac{x^3+5}{2x+1}$. Find the HA.

By general rule,

$$\lim_{x \to -\infty} h(x) = \lim_{x \to -\infty} \frac{x^3}{2x} = \lim_{x \to -\infty} \frac{x^2}{2} = \infty \rightarrow$$

$$\lim_{x \to \infty} h(x) = \lim_{x \to \infty} \frac{x^3}{2x} = \lim_{x \to \infty} \frac{x^2}{2} = \infty \rightarrow$$

No HA