

Lesson 22: Limits at Infinity Pt 2

Slant Asymptotes

The line $y = ax + b$, where a and b are constants and $a \neq 0$, is a slant asymptote of $f(x)$ if $f(x)$ gets closer and closer to $y = ax + b$ as $x \rightarrow \pm\infty$.

To find the slant asymptotes of a rational function, we use

- Synthetic Division
- Long Division

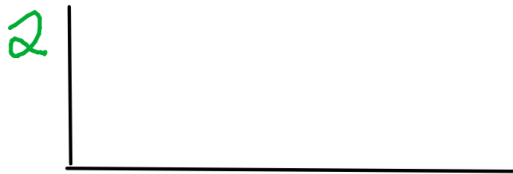
Synthetic Division Slide 1

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 1: Synthetic Division

To use Synthetic Division, find the zeros of the denominator of y .

$$\text{i.e. } x - 2 = 0 \implies x = 2$$



Synthetic Division Slide 2

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 1: Synthetic Division

Next, list the coefficients of the polynomial on the numerator.

$$\begin{array}{r|rrrr} 2 & 1 & 2 & 8 & \end{array}$$

Synthetic Division Slide 3

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 1: Synthetic Division

Drag down the first # inside the \perp

$$\begin{array}{r|rrrr} 2 & 1 & 2 & 8 & \\ & \downarrow & & & \\ & 1 & & & \end{array}$$

Synthetic Division Slide 4

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 1: Synthetic Division

Multiply 2 by the value just written on the bottom row.

$$\begin{array}{r|rrr}
 2 & 1 & 2 & 8 \\
 & \downarrow & 2 & \\
 \hline
 & & &
 \end{array}$$

Synthetic Division Slide 5

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 1: Synthetic Division

Add the column created.

$$\begin{array}{r|rrr}
 2 & 1 & 2 & 8 \\
 & \downarrow & 2 & \downarrow \\
 \hline
 & 1 & 4 &
 \end{array}$$

Synthetic Division Slide 6

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 1: Synthetic Division

Repeat: Multiply 2 by the value just written on the bottom row.

$$\begin{array}{r|rrr}
 2 & 1 & 2 & 8 \\
 & \downarrow & 2 & 8 \\
 \hline
 & 1 & 4 & 8
 \end{array}$$

Synthetic Division Slide 7

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 1: Synthetic Division

Repeat: Add the column created.

$$\begin{array}{r|rrr}
 2 & 1 & 2 & 8 \\
 & \downarrow & 2 & 8 \\
 \hline
 & 1 & 4 & 16
 \end{array}$$

Synthetic Division Slide 8

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 1: Synthetic Division

Rewrite $y = \frac{x^2 + 2x + 8}{x - 2} = x + 4 + \frac{16}{x - 2}$

2	1	2	8	.
	↓	2	8	↓
	1	4	16	↙ remainder

Synthetic Division Slide 9

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 1: Synthetic Division

Rewrite $y = \frac{x^2 + 2x + 8}{x - 2} = \boxed{x + 4} + \frac{16}{x - 2}$

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Slant Asymptote

i.e. Slant Asymptote $y = x + 4$

Long Division Slide 1

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 2: Long Division

Long Division works w/o knowing the zeros of y 's denominator. But involves a lot of work.

$$\underbrace{x - 2}_{\text{Divisor}} \sqrt{\underbrace{x^2 + 2x + 8}_{\text{Dividend}}}$$

Long Division Slide 2

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 2: Long Division

Divide the first term of the dividend by first term of the divisor.

$$x - 2 \sqrt{x^2 + 2x + 8}$$

$$\textcircled{1} \frac{x^2}{x} = x$$

Long Division Slide 3

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 2: Long Division

Place what's found in ① above $\sqrt{\quad}$

$$x - 2 \overline{\begin{array}{r} x \\ x^2 + 2x + 8 \end{array}}$$

$$\textcircled{1} \frac{x^2}{x} = x$$

Long Division Slide 4

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 2: Long Division

Multiply that term in ① with divisor.

$$x - 2 \overline{\begin{array}{r} x \\ x^2 + 2x + 8 \end{array}}$$

$$\textcircled{1} \frac{x^2}{x} = x$$

$$\textcircled{2} x(x - 2) = x^2 - 2x$$

Long Division Slide 5

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 2: Long Division

Line up what you found in ① under the dividend

$$x-2 \overline{) \begin{array}{r} x^2 + 2x + 8 \\ -(x^2 - 2x) \\ \hline \end{array}}$$

$$\textcircled{1} \frac{x^2}{x} = x$$

$$\textcircled{2} x(x-2) = x^2 - 2x$$

Long Division Slide 6

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 2: Long Division

Subtract and bring down the next term.

$$x-2 \overline{) \begin{array}{r} x^2 + 2x + 8 \\ -(x^2 - 2x) \quad \downarrow \\ \hline 4x + 8 \end{array}}$$

$$\textcircled{1} \frac{x^2}{x} = x$$

$$\textcircled{2} x(x-2) = x^2 - 2x$$

Long Division Slide 7

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 2: Long Division

Repeat: Divide the first term of the dividend by first term of the divisor.

$$\begin{array}{r}
 x \\
 x-2 \overline{) x^2 + 2x + 8} \\
 \underline{-(x^2 - 2x)} \\
 4x + 8
 \end{array}$$

$$\textcircled{1} \frac{4x}{x} = 4$$

Long Division Slide 8

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 2: Long Division

Repeat: Place what's found in $\textcircled{1}$ above $\sqrt{}$

$$\begin{array}{r}
 x + 4 \\
 x-2 \overline{) x^2 + 2x + 8} \\
 \underline{-(x^2 - 2x)} \\
 4x + 8
 \end{array}$$

$$\textcircled{1} \frac{4x}{x} = 4$$

Long Division Slide 9

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 2: Long Division

Repeat: Multiply that term in ① with divisor.

$$\begin{array}{r}
 x+4 \\
 x-2 \overline{) x^2 + 2x + 8} \\
 \underline{-(x^2 - 2x)} \quad \downarrow \\
 4x + 8
 \end{array}$$

$$\textcircled{1} \frac{4x}{x} = 4$$

$$\textcircled{2} 4(x-2) = 4x - 8$$

Long Division Slide 10

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 2: Long Division

Repeat: Line up what you found in ② under the dividend

$$\begin{array}{r}
 x+4 \\
 x-2 \overline{) x^2 + 2x + 8} \\
 \underline{-(x^2 - 2x)} \quad \downarrow \\
 4x + 8 \\
 \underline{-(4x - 8)} \\
 16
 \end{array}$$

$$\textcircled{1} \frac{4x}{x} = 4$$

$$\textcircled{2} 4(x-2) = 4x - 8$$

Long Division Slide 11

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 2: Long Division

Repeat: Subtract.

$$\begin{array}{r}
 x-2 \overline{) x^2 + 2x + 8} \\
 \underline{-(x^2 - 2x)} \quad \downarrow \\
 4x + 8 \\
 \underline{-(4x - 8)} \\
 16
 \end{array}$$

$$\textcircled{1} \frac{4x}{x} = 4$$

$$\textcircled{2} 4(x-2) = 4x - 8$$

Long Division Slide 12

Ex 5: Find the slant asymptote of $y = \frac{x^2 + 2x + 8}{x - 2}$

Method 2: Long Division

Rewrite $y = \frac{x^2 + 2x + 8}{x - 2}$

$$= \boxed{x + 4} + \frac{16}{x - 2}$$

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Slant Asymptote

i.e. Slant Asymptote $y = x + 4$

$$\begin{array}{r}
 x-2 \overline{) x^2 + 2x + 8} \\
 \underline{-(x^2 - 2x)} \quad \downarrow \\
 4x + 8 \\
 \underline{-(4x - 8)} \\
 16
 \end{array}$$

Ex 6: Find the slant asymptote of

$$y = \frac{x^3 + x^2 + 1}{x+3} = \frac{x^3 + x^2 + 0x + 1}{x+3}$$

Synthetic Division: zeros of $x+3 \Rightarrow x = -3$

$$\begin{array}{r|rrrr} -3 & 1 & 1 & 0 & 1 \\ & \downarrow & -3 & 6 & -18 \\ \hline & 1 & -2 & 6 & -17 \end{array}$$

$$g(x) = \boxed{x^2 - 2x + 6} - \frac{17}{x+3}$$

But $g(x)$ has no slant asymptote

Recall a slant asymptote has the form $y = ax + b$.

Is $y = x^2 - 2x + 6$ of that form? No

Hence no slant asymptote.

Note: You can avoid all that work by checking the difference b/w the power of the leading terms of numerator and denominator.