Lesson 27: Antiderivatives and Indefinite Integration

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Consider the equation $F'(x) = f(x)$.

Two ways to interpret this:

① $f(x)$ = derivative of $F(x)$

② $F(x)$ = antiderivative of $f(x)$

Notation: $F(x) = \int f(x)\,dx$

With antiderivatives start with $f(x)$ and find $F(x)$.

Ex 1: ⓐ Differentiate $F(x) = x^2 + 2$. $F'(x) = 2x$

ⓑ Find $\int 2x\,dx$.

What function $F(x)$ has $2x$ as its derivative?

By ⓐ, one such $F(x)$ is x^2+2.

But so are:

- x^2
- $x^2 - 1234$
- $x^2 +$ (constant)

Why? Derivative of a constant is zero.

To account for this, use C as an arbitrary constant:

$$\int 2x\,dx = x^2 + C$$

Process of finding all the antiderivatives of a function is called $\boxed{\text{indefinite integration.}}$

Denoted by $\underbrace{\int f(x)\,dx}= F(x) + C$ where C is a constant

Read as "integral of f(x)"

- $\int$ integral sign
- $f(x)$ integrand
- x integration variable
- C constant of integration

Differentiation Rule	Integration Rule
$\frac{d}{dx}(c) = 0$	$\int 0\,dx = c$
$\frac{d}{dx}(kx) = k$	$\int k\,dx = kx + c$
$\frac{d}{dx}(kf(x)) = kf'(x)$	$\int kf'(x)\,dx = k\int f'(x)\,dx$ $= kf(x) + C$
$\frac{d}{dx}(x^n) = nx^{n-1}$ $\frac{d}{dx}(x^{n+1}) = (n+1)x^n$	$\int nx^{n-1}\,dx = x^n + c$ $\int (n+1)x^n\,dx = x^{n+1} + c$ $(n+1)\int x^n\,dx = x^{n+1} + c$ $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

Ex 1: Find the indefinite integral

$$\int (x^2 + 2\sqrt{x})\,dx$$

$$\int (x^2 + 2x^{1/2})\,dx = \frac{x^{2+1}}{2+1} + 2\frac{x^{1/2+1}}{1/2+1} + C$$

$$= \frac{x^3}{3} + 2\frac{x^{3/2}}{3/2} + C$$

$$= \frac{x^3}{3} + 2\cdot\frac{2}{3}x^{3/2} + C$$

$$= \frac{x^3}{3} + \frac{4}{3}x^{3/2} + C$$

Differentiation Rule	Integration Rule
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$

$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\int \csc x \cot x\,dx = -\csc x + C$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}(\ln x) = \frac{1}{x},\ x > 0$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$

Always take the derivative of your answer!!!

Especially when you have trig functions!!!

Ex 2: Evaluate $\int \frac{\sin x + \cos x}{2} dx$

$$\frac{1}{2}\int (\sin x + \cos x) dx$$

$$= \frac{1}{2}\left[\underbrace{\int \sin x dx}_{-\cos x} + \underbrace{\int \cos x dx}_{\sin x}\right]$$

$$= \frac{1}{2}(-\cos x + \sin x) + C$$

Ex 3: Evaluate $\int x^e + \frac{1}{x} + 1 \quad dx$

$$\int x^e + \frac{1}{x} + 1 \, dx = \int x^e dx + \int \frac{1}{x} dx + \int dx$$

$$= \frac{x^{e+1}}{e+1} + \ln|x| + x + C$$