Lesson 28: More on Antiderivatives and Indefinite Integration

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A differential eqn in x and y is an eqn that relates x, y, and y'.

examples: $y' = 3x$

$x + y' = 5$

$2y + xy = e^x$

Ex 1: Solve the differential eqn $y' = 3x$

$$\int y'\,dx = \int 3x\,dx$$

$$\int \frac{dy}{\cancel{dx}}\,\cancel{dx} = \int 3x\,dx$$

$$\int dy = \int 3x\,dx$$

$$y = \frac{3x^2}{2} + C$$

Recall

$$y' = \frac{dy}{dx}$$

This solution is also called general solution.

- But what if I give you a y-value at some x?

  We call that the initial condition.

- The answer of a specific function is particular solution

- A differential eqn w/ initial condition is an initial value problem (IVP).

Ex 2: Solve IVP $y' = 3x$ with $y(0) = 2$

By Ex 3, $y = \frac{3}{2}x^2 + C$

So $2 = y(0) = \frac{3}{2}(0)^2 + C$

$2 = 0 + C$

$2 = C$

Answer: $y = \frac{3}{2}x^2 + 2$

Ex 3: Solve the IVP

$$2y'' = e^x + 4 \qquad y'(0) = 5 \qquad y(2) = 10$$

Solve for y''

$$y'' = \frac{e^x + 4}{2} = \frac{e^x}{2} + \frac{4}{2} = \frac{e^x}{2} + 2$$

Integrate each side

$$\int y'' \, dy = \int \left(\frac{e^x}{2} + 2\right) dx$$

$$y' = \frac{e^x}{2} + 2x + C$$

Ex 3: Solve the IVP

$$2y'' = e^x + 4 \qquad \boxed{y'(0) = 5} \qquad y(2) = 10$$

Plug the condition into y'. To find C.

$$y' = \frac{e^x}{2} + 2x + C$$

$$5 = y'(0) = \frac{e^0}{2} + 2(0) + C$$

$$5 = \frac{1}{2} + C$$

$$C = 5 - \frac{1}{2} = \frac{9}{2}$$

So $y' = \frac{e^x}{2} + 2x + \frac{9}{2}$

Ex 3: Solve the IVP

$$2y'' = e^x + 4 \qquad y'(0) = 5 \qquad y(2) = 10$$

Repeat: Integrate each side

$$y' = \frac{e^x}{2} + 2x + \frac{9}{2}$$

$$\int y'\,dy = \int \frac{e^x}{2} + 2x + \frac{9}{2}\,dx$$

$$y = \frac{e^x}{2} + \frac{2x^2}{2} + \frac{9}{2}x + C$$

$$= \frac{e^x}{2} + x^2 + \frac{9}{2}x + C$$

Ex 3: Solve the IVP

$$2y'' = e^x + 4 \qquad y'(0) = 5 \qquad \boxed{y(2) = 10}$$

Repeat: Plug the condition into y'. To find C.

$$y = \frac{e^x}{2} + x^2 + \frac{9}{2}x + C$$

$$10 = y(2) = \frac{e^2}{2} + 2^2 + \frac{9}{\cancel{2}}\cancel{(2)} + C$$

$$10 = \frac{e^2}{2} + 4 + 9 + C$$

$$C = 10 - \frac{e^2}{2} - 4 - 9 = -\frac{e^2}{2} - 3$$

So $\boxed{y = \frac{e^x}{2} + x^2 + \frac{9}{2}x - \frac{e^2}{2} - 3}$