Lesson 30: Definite Integrals

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When the # of rectangles used gets bigger and bigger, the approx gets better and better.

i.e. the approx gets closer and closer to the exact signed area

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What happens when $n \to \infty$?

Left/Right Riemann Sum approaches the actual signed area.

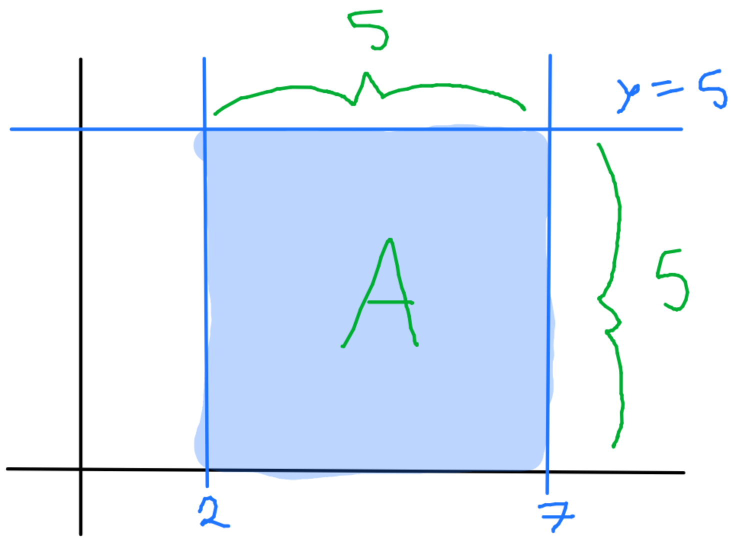
Signed Area $= \int_a^b f(x)dx$

a – lower limit of integration
b – upper limit of integration

An integral w/ lower and upper limits is called a definite integral.

So no more +C, when you see lower/upper limits (i.e. a & b)

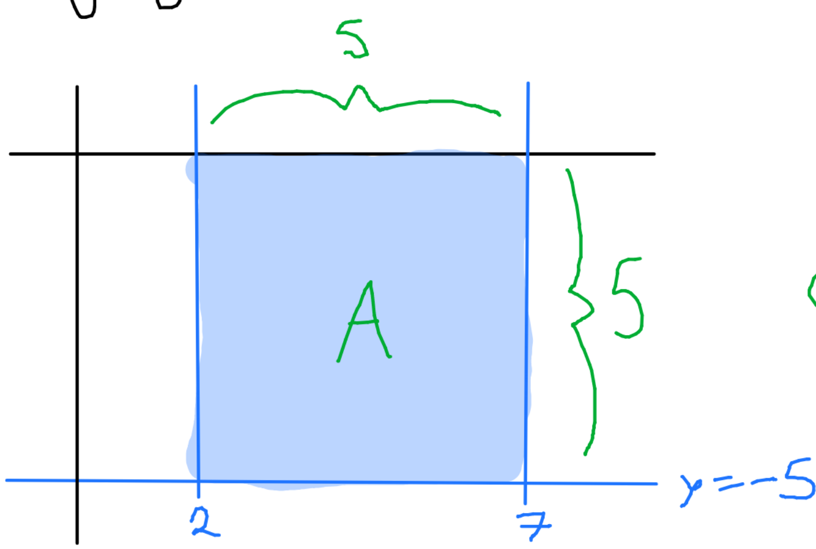
Ex 1: Evaluate the definite integral $\int_2^7 5\,dx$ by using geometric formulas.



$A = 5 \times 5 = 25$

$\int_2^7 5\,dx = 25$

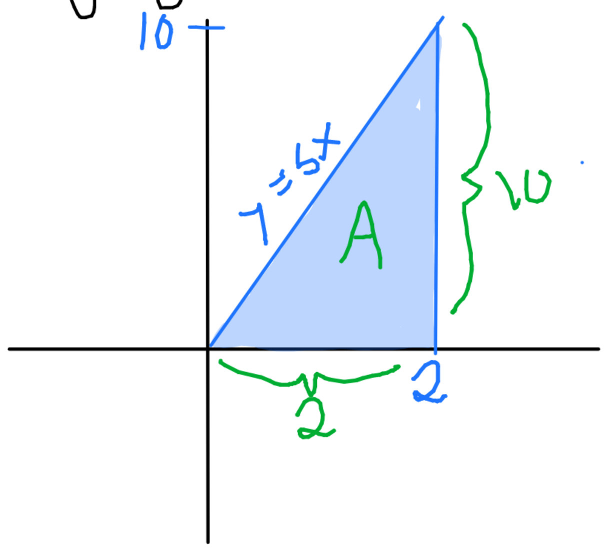
Ex 2: Evaluate the definite integral $\int_2^7 -5\,dx$ by using geometric formulas.



$A = 25$

$\int_2^7 -5\,dx = -25$

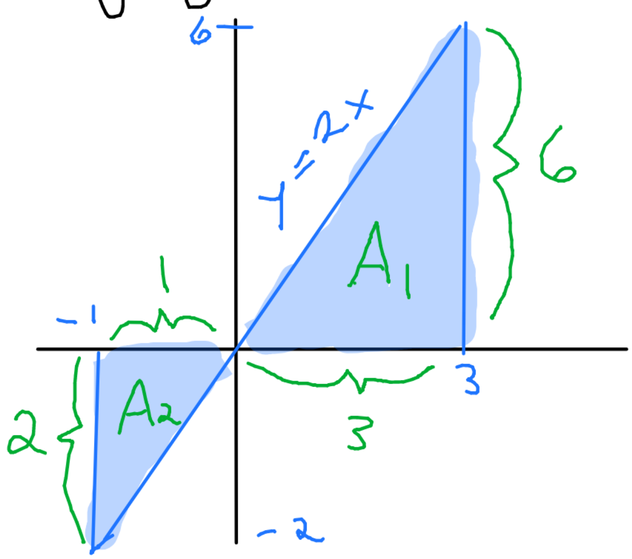
Ex 3: Evaluate the definite integral $\int_0^2 5x\,dx$ by using geometric formulas.



$A = \frac{1}{2}(2)(10) = 10$

$\int_0^2 5x\,dx = 10$

Ex 4: Evaluate the definite integral $\int_{-1}^3 2x\,dx$ by using geometric formulas.

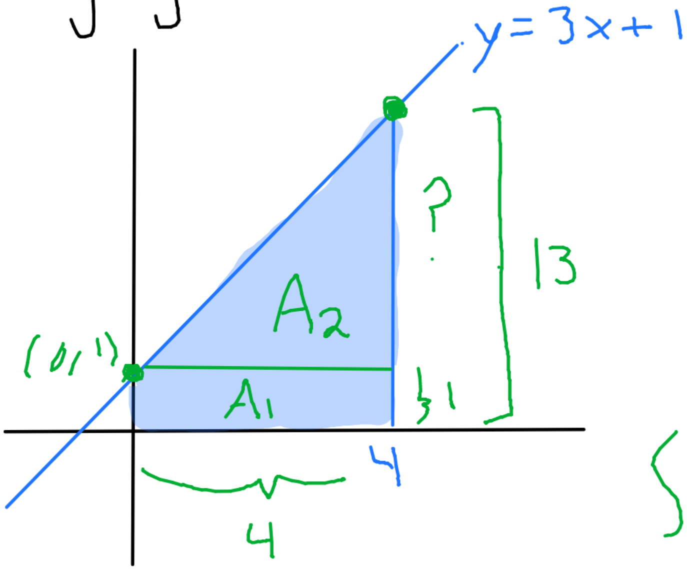


$A_1 = \frac{1}{\cancel{2}}(3)(\cancel{6}^{3}) = 9$

$A_2 = \frac{1}{2}(1)(2) = 1$

$$\int_{-1}^3 2x\,dx = A_1 - A_2 = 9 - 1 = 8$$

Ex 5: Evaluate the definite integral $\int_0^4 (3x+1)\,dx$ by using geometric formulas.



$A_1 = 1(4) = 4$

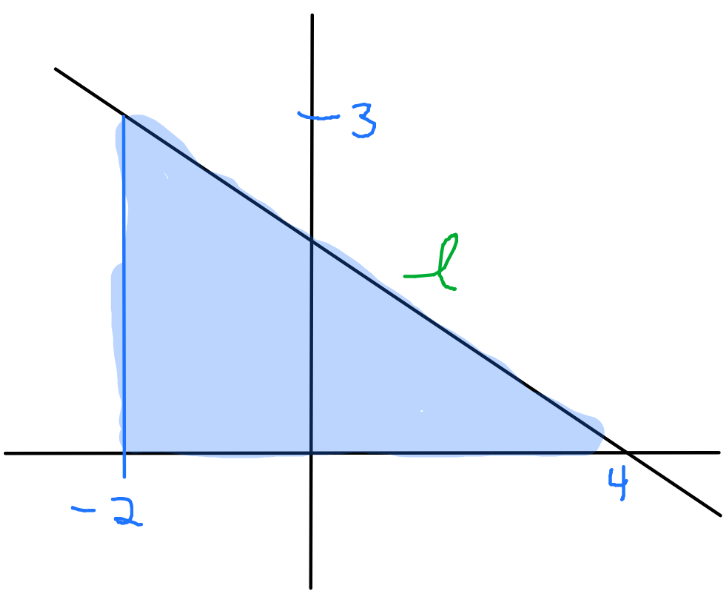
$y = 3(4) + 1 = 13$

$? = 13 - 1 = 12$

$A_2 = \frac{1}{2}(4)(12) = 24$

$\int_0^4 (3x+1)\,dx = 24 + 4 = 28$

Ex 6: Write the definite integral that represents the shaded area below.



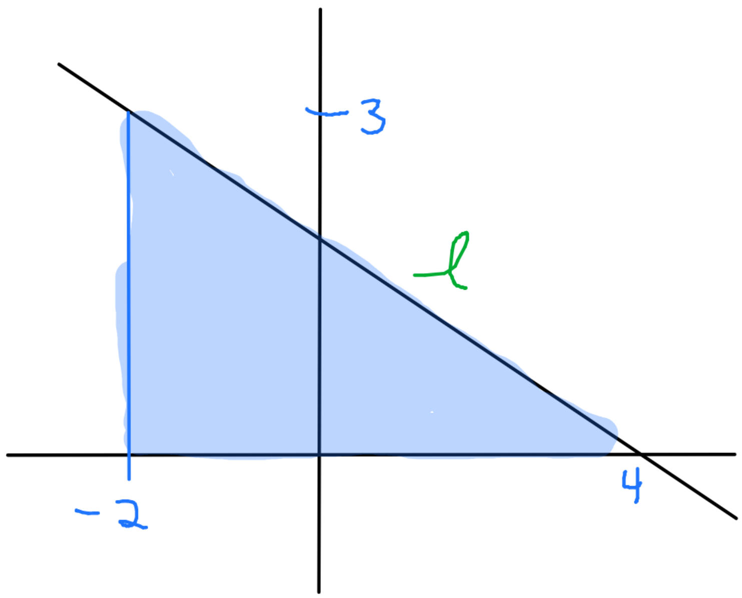
$\int_{-2}^{4} \boxed{}\, dx$

$(-2, 3)$ & $(4, 0)$

$m = \frac{0-3}{4-(-2)} = \frac{-3}{6} = \frac{-1}{2}$

$y = -\frac{1}{2}x + b$

Ex 6: Write the definite integral that represents the shaded area below.



$$\boxed{\int_{-2}^{4} \left(-\frac{1}{2}x+2\right) dx}$$

$(-2, 3)$ & $\boxed{(4, 0)}$

$y = -\frac{1}{2}x + b$

$0 = -\frac{1}{2}(4) + b$

$0 = -2 + b$

$b = 2$