Lesson 31: Definite Integrals Pt 2

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Properties of Definite Integrals

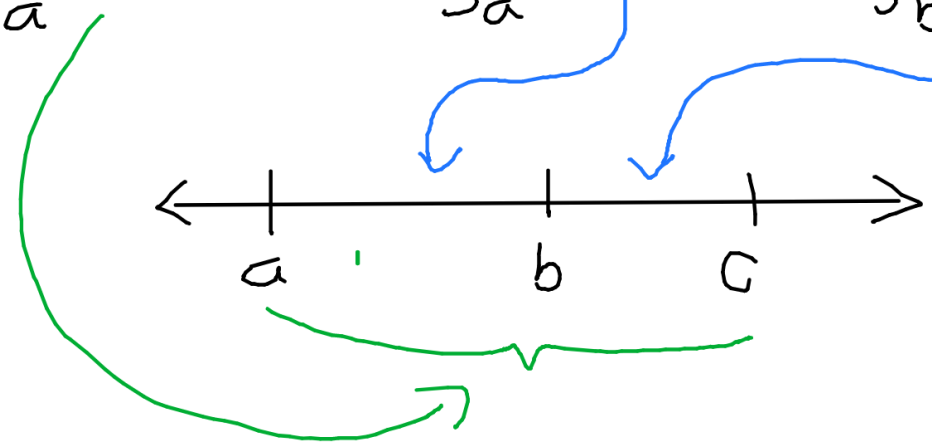
$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$



Ex 1: Given $\int_1^3 f(x)\,dx = 5$, $\int_3^4 f(x)\,dx = 2$ and $\int_1^3 g(x)\,dx = 10$. Evaluate the following

(a) $\int_1^3 2f(x)\,dx = 2\underbrace{\int_1^3 f(x)\,dx}_{=5} = 2(5) = 10$

Ex 1: Given $\int_1^3 f(x)\,dx = 5$, $\int_3^4 f(x)\,dx = 2$ and $\int_1^3 g(x)\,dx = 10$. Evaluate the following

(b) $\int_4^3 f(x)\,dx = -\int_3^4 f(x)\,dx = -2$

Ex 1: Given $\int_1^3 f(x)\,dx = 5$, $\int_3^4 f(x)\,dx = 2$ and $\int_1^3 g(x)\,dx = 10$. Evaluate the following

(c) $\int_1^3 [2f(x) - 3g(x)]\,dx$

$= 2\underbrace{\int_1^3 f(x)\,dx}_{=5} - 3\underbrace{\int_1^3 g(x)\,dx}_{=10}$

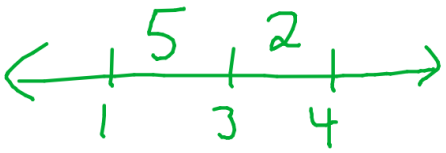
$= 2(5) - 3(10)$

$= 10 - 30 = -20$

Ex 1: Given $\int_1^3 f(x)\,dx = 5$, $\int_3^4 f(x)\,dx = 2$ and $\int_1^3 g(x)\,dx = 10$. Evaluate the following

(d) $\int_1^4 f(x)\,dx = \int_1^3 f(x)\,dx + \int_3^4 f(x)\,dx$

$= 5 + 2 = 7$



Ex 2: Given $\int_3^7 x^2\,dx = \frac{316}{3}$, $\int_3^7 x\,dx = 20$, and $\int_3^7 dx = 4$, evaluate

$\int_3^7 [-4x^2 + x - 8]\,dx$

$= -4\underbrace{\int_3^7 x^2\,dx}_{316/3} + \underbrace{\int_3^7 x\,dx}_{20} - 8\underbrace{\int_3^7 dx}_{4}$

$= -4\left(\frac{316}{3}\right) + 20 - 8(4) = -\frac{1300}{3}$

Ex 3: Given $\int_2^6 2x^3\,dx = 640$. Find

$$\int_2^6 8x^3\,dx = \int_2^6 4(2x^3)\,dx$$

$$= 4\int_2^6 2x^3\,dx$$

$$= 4(640)$$

$$= 2560$$

Ex 4: Given $\int_a^b g(x)\,dx = 5$ and $\int_a^c g(x)\,dx = 8\int_a^b g(x)\,dx$. Compute $\int_b^c g(x)\,dx$

Recall $\int_a^c g(x)\,dx = \int_a^b g(x)\,dx + \boxed{\int_b^c g(x)\,dx}$

Solve for the green box.

$$\int_b^c g(x)\,dx = \int_a^c g(x)\,dx - \int_a^b g(x)\,dx$$

$$= 8\int_a^b g(x)\,dx - \int_a^b g(x)\,dx$$

$$= 7\int_a^b g(x)\,dx = 7(5) = 35$$