Lesson 33: The Fundamental Theorem of Calculus (FTC) Pt 2

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Recall $\int_a^b f(x)\,dx = F(x)\Big]_a^b = F(b) - F(a)$

Question: $\int_a^b g'(x)\,dx = ?$

Well the antiderivative of $g'(x)$ is $g(x)$.

So $\int_a^b g'(x)\,dx = g(x)\Big]_a^b = g(b) - g(a)$

Ex 1: Compute $\int_0^{\pi} (\cos x \tan x - \sec^2 x)\,dx$

$$= \int_0^{\pi} \left(\frac{\cancel{\cos x}}{1} \cdot \frac{\sin x}{\cancel{\cos x}} - \sec^2 x\right) dx$$

$$= \int_0^{\pi} (\sin x - \sec^2 x)\,dx$$

$$= (-\cos x - \tan x)\Big]_0^{\pi}$$

$$= -\cos(\pi) - \tan(\pi) - (-\cos(0) - \tan(0))$$

$$= -(-1) - 0 - (-1 - 0)$$

$$= 1 + 1$$

$$= 2$$

Ex 2: The growth rate of the population of a city is

$$p'(t) = -500(3-t)$$

where t is time in years. How does the population change t = 1 year to t = 3 years?

$$\int_1^3 p'(t)\,dt = \int_1^3 -500(3-t)\,dt$$

$$= -500\int_1^3 (3-t)\,dt$$

$$= -500\left(3t - \frac{t^2}{2}\right)\Big]_1^3$$

$$= -500\left[\left(3(3) - \frac{3^2}{2}\right) - \left(3(1) - \frac{1^2}{2}\right)\right]$$

$$= -500\left[9 - \frac{9}{2} - 3 + \frac{1}{2}\right]$$

$$= -500\left[6 - \frac{8}{2}\right]$$

$$= -500(6-4)$$

$$= -1000$$

Recall

- Displacement is the difference in position.
- It could be positive or negative
- The sign indicates the direction.

By FTC,

$$\int_a^b v(t)\,dt = \int_a^b s'(t)\,dt = s(t)\Big]_a^b$$
$$= s(b) - s(a)$$

Ex 3: The velocity function, in feet per second, is given for a particle moving along a straight line

$$v(t) = -10t + 20$$

where t is in seconds.

(a) Find the displacement from t = 0 to t = 2 seconds.

$$\int_0^2 v(t)\,dt = \int_0^2 (-10t+20)\,dt$$
$$= \left(\frac{-10t^2}{2} + 20t\right)\Big]_0^2$$
$$= (-5t^2 + 20t)\Big]_0^2$$
$$= (-5(2)^2 + 20(2)) - \xrightarrow{0}(\cancel{-5(0)^2 + 20(0)})$$
$$= -20 + 40 = 20$$

Ex 3: The velocity function, in feet per second, is given for a particle moving along a straight line

$$v(t) = -10t + 20$$

where t is in seconds.

(b) Find the displacement from t = 0 to t = 4 seconds.

$$\int_0^4 v(t)\,dt = (-5t^2 + 20t)\Big]_0^4$$ → from your work in part (a)
$$= -5(4)^2 + 20(4) - 0$$
$$= -80 + 80$$
$$= 0$$

HW 33.4: A faucet is turned on at 9:00 am and water starts to flow into a tank at the rate of

$$r(t) = 6\sqrt{t}$$

where t is time in hours after 9:00 am and the rate r(t) is in cubic feet per hour.

(a) How much water, in cubic feet, flows into the tank from 10:00 am to 1:00 pm?

9:00am $\Rightarrow t=0$
10:00am $\Rightarrow t=1$
1:00pm $\Rightarrow t=4$

$$\int_1^4 r(t)\,dt = \int_1^4 6t^{1/2}\,dt$$

$$= \frac{6t^{3/2}}{3/2}\Big]_1^4$$

$$= \frac{2}{3}\cdot 6t^{3/2}\Big]_1^4$$

$$= 4t^{3/2}\Big]_1^4$$

$$= 4(4)^{3/2} - 4(1)^{3/2}$$

$$= 4\cdot 2^3 - 4\cdot 1$$

$$= 32 - 4$$

$$= 28$$

HW 33.4: A faucet is turned on at 9:00 am and water starts to flow into a tank at the rate of

$$r(t) = 6\sqrt{t}$$

where t is time in hours after 9:00 am and the rate r(t) is in cubic feet per hour.

(b) How many hours after 9:00 am will there be 121 cubic feet of water in the tank?

$\int_0^x r(t)\,dt = 121$ solve this for x.

By using my work in (a).

$$4t^{3/2}\Big]_0^x = 121$$

$$4x^{3/2} - \cancel{4(0)^{3/2}} \xrightarrow{} 0 = 121$$

$$4x^{3/2} = 121$$

$$x^{3/2} = 121/4$$

$$\left(x^{3/2}\right)^{2/3} = (121/4)^{2/3}$$

$$x = (121/4)^{2/3}$$