Lesson 4: Finding Limits Analytically

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There are 3 different cases to consider..

① $f(c)$ returns a # (it could be 0)

i.e. $f(x)$ is continuous @ $x=c$

i.e. $\lim_{x\to c} f(x) = f(c)$

Ex 1: $\lim_{x\to 4} (2x-3) = 2(4)-3 = 8-3 = 5$

② $f(c)$ returns $\dfrac{\text{nonzero \#}}{0}$

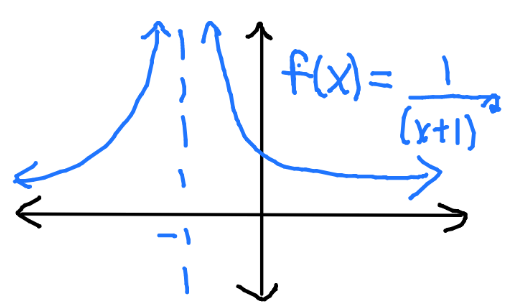
i.e. Vertical Asymptote @ $x=c$

i.e. $\lim_{x\to c} f(x) = \pm\infty$ or DNE

Ex 2: $\lim_{x\to -1} \dfrac{1}{(x+1)^2}$

$f(-1) = \dfrac{1}{(-1+1)^2} = \dfrac{1}{0} \Rightarrow$ We need to check the left and right limits.

$$\begin{cases} \lim_{x\to -1^-} \dfrac{1}{(x+1)^2} = \infty \\ \lim_{x\to -1^+} \dfrac{1}{(x+1)^2} = \infty \end{cases}$$



$$\lim_{x\to -1} \frac{1}{(x+1)^2} = \infty$$

Ex 3: $\displaystyle\lim_{x\to -1} \frac{-1}{(x+1)^2} = -\infty$

Ex 4: $\displaystyle\lim_{x\to 0} \frac{1}{x}$

$f(0) = \frac{1}{0} \Rightarrow$ We need to check the left and right limits.

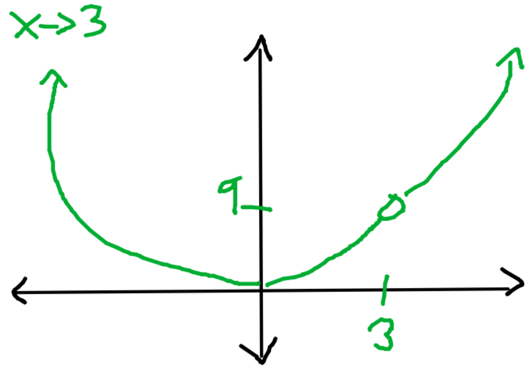
But remember we have already determined the value of this limit.

$$\lim_{x\to 0} \frac{1}{x} = DNE$$

Ex 5: $\displaystyle\lim_{x\to 3} \frac{x^3-3x^2}{x-3}$

$f(3) = \frac{3^3 - 3\cdot 3^2}{3-3} = \frac{27-27}{3-3} = \frac{0}{0} \Rightarrow$ Let's try factoring

$$\lim_{x\to 3} \frac{x^3-3x^2}{x-3} = \lim_{x\to 3} \frac{x^2\cancel{(x-3)}}{\cancel{x-3}} = \lim_{x\to 3} x^2 = 3^2 = \boxed{9}$$



Ex 6: $\lim_{x\to 2} \frac{x^2-2x}{(x-2)^2}$

$f(2) = \frac{2^2-2\cdot 2}{(2-2)^2} = \frac{4-4}{(2-2)^2} = \frac{0}{0} \Rightarrow$ Let's try factoring

$\lim_{x\to 2} \frac{x^2-2x}{(x-2)^2} = \lim_{x\to 2} \frac{x\cancel{(x-2)}}{(x-2)^{\cancel{2}}} = \lim_{x\to 2} \frac{x}{x-2} = \frac{2}{0}$

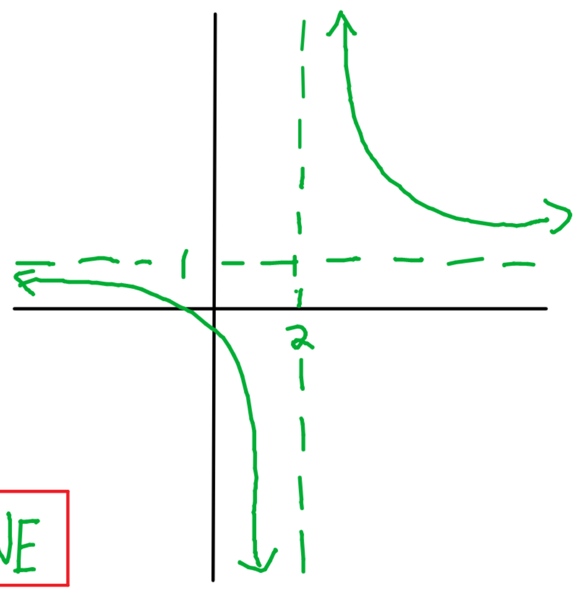
But now it looks like Case 2. So we need to check the left and right limits.

Note that

$\frac{x}{x-2} = \frac{x-2+2}{x-2}$

$= \frac{x-2}{x-2} + \frac{2}{x-2}$

$= 1 + \frac{2}{x-2}$



Hence by the graph

$\lim_{x\to 2} \frac{x^2-2x}{(x-2)^2} = \lim_{x\to 2} \frac{x}{x-2} = \boxed{\text{DNE}}$

Ex 7: $f(x) = \begin{cases} 1/x & \text{if } x \geq -2 \\ -x & \text{if } x < -2 \end{cases}$

(a) $\lim_{x\to -2} f(x) = \text{DNE}$

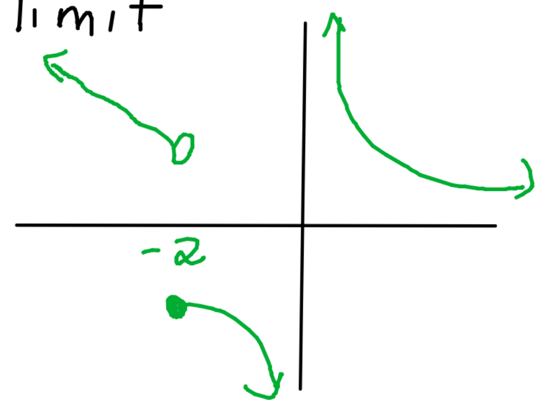
Notice the function splits @ $x=-2$. So we need to check left/right limits.

$$\lim_{x\to -2^-} f(x) = \lim_{x\to -2} (-x) = -(-2) = 2$$
$$\lim_{x\to -2^+} f(x) = \lim_{x\to -2} \left(\frac{1}{x}\right) = \frac{1}{-2} = -\frac{1}{2}$$

Ex 7: $f(x) = \begin{cases} 1/x & \text{if } x \geq -2 \\ -x & \text{if } x < -2 \end{cases}$

(b) $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{1}{x} = \text{DNE}$

By now we know this limit



Ex 8: $f(x) = \begin{cases} \sin x & \text{if } x \geq 0 \\ x^2 & \text{if } x < 0 \end{cases}$

Find $\lim_{x\to 0} f(x)$

Notice the function splits @ $x = 0$. So we need to check left/right limits.

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0} (x^2) = 0$$
$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} (\sin x) = 0$$
$$\Rightarrow \lim_{x\to 0} f(x) = 0$$