Lesson 5: Continuity

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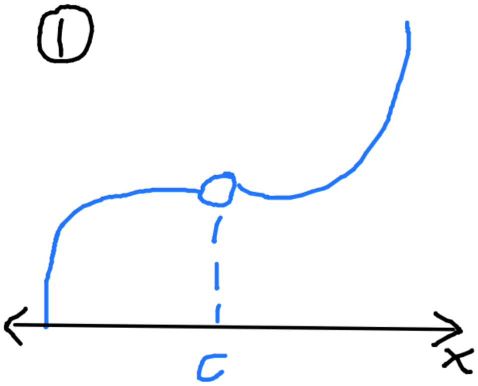
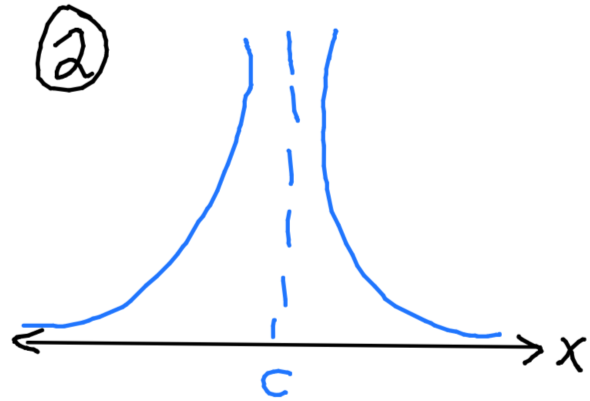
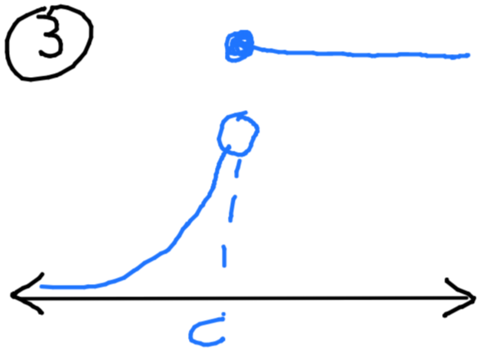
A function is continuous if there is no disruption in the graph.

The following 4 graphs show f(x) is discontinuous @ x = c.

① 

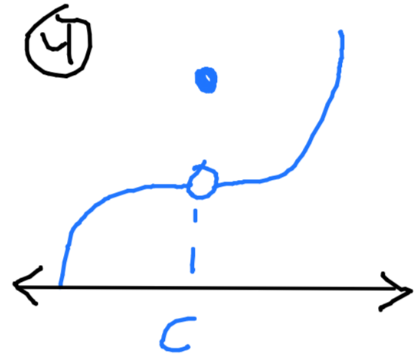
② 

These 2 graphs have f(c) undefined.

③ 

f(c) is defined

BUT $\lim_{x \to c} f(x) = \text{DNE}$

④ 

f(c) is defined

$\lim_{x \to c} f(x)$ exists

BUT $\lim_{x \to c} f(x) \neq f(c)$

We can see a function $f(x)$ is continuous @ $x=c$ if the following is true:

① $f(c)$ defined (has a value)

② $\lim_{x \to c} f(x)$ exists

③ $\lim_{x \to c} f(x) = f(c)$

If any of the 3 conditions aren't met, then we say $f(x)$ is discontinuous @ $x=c$.

Ex 1: Discuss continuity of $f(x) = \frac{2x}{x^2-x}$

When is $f(x)$ undefined? When denominator $=0$

$x^2 - x = 0$

$x(x-1) = 0$

$x=0$ | $x-1=0$

$x=1$

Hence we have discontinuities @ $x=0,1$

But what kind of discontinuity are they?

Jump? Hole? VA?

Let's simplify to answer that question.

$$f(x) = \frac{2x}{x^2-x} = \frac{\cancel{2x}}{\cancel{x}(x-1)} = \frac{2}{x-1}$$

What factor canceled? x $\Rightarrow x=0$ Hole

What factor remains? $x-1$ $\Rightarrow x=1$ VA

(in the denominator)

HW 5.4: Discuss continuity of $f(x)=\dfrac{x^2+2x-3}{x^2+5x-6}$

When is $f(x)$ undefined? When denominator $=0$

$$x^2+5x-6=0$$
$$x^2-x+6x-6=0$$
$$x(x-1)+6(x-1)=0$$
$$(x+6)(x-1)=0$$

$x+6=0$ | $x-1=0$
$x=-6$ | $x=1$

-6
$\wedge$
$-1+6=5$

Hence we have discontinuities @ $x=-6, 1$

But what kind of discontinuity are they? Jump? Hole? VA?

Let's simplify to answer that question.

$$x^2+2x-3$$
$$x^2-x+3x-3$$
$$x(x-1)+3(x-1)$$
$$(x+3)(x-1)$$

-3
$\wedge$
$2=-1+3$

So $f(x)=\dfrac{x^2+2x-3}{x^2+5x-6}=\dfrac{\cancel{(x-1)}(x+3)}{\cancel{(x-1)}(x+6)}=\dfrac{x+3}{x+6}$

What factor canceled? $\underline{x-1}$ $\Rightarrow x=1$ Hole

What factor remains? $\underline{x+6}$ $\Rightarrow x=-6$ VA

(in the denominator)

<u>Ex2</u>: Discuss continuity of

$$f(x)=\begin{cases} e^{-x} & \text{if } x\le 0 \\ \sqrt{x}-1 & \text{if } x>0 \end{cases}$$

To determine continuity, we need to check

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0} e^{-x} = e^{-0} = 1$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} (\sqrt{x}-1) = 0-1 = -1$$

Discontinuity @ $x=0$

$\Downarrow$

Jump $x=0$

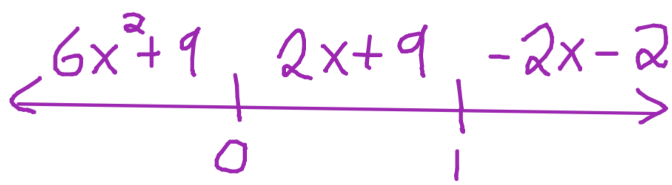
<u>HW 5.8</u>: Discuss continuity of

$$f(x)=\begin{cases} 6x^2+9 & \text{if } x\le 0 \\ 2x+9 & \text{if } 0<x<1 \\ -2x-2 & \text{if } x\ge 1 \end{cases}$$

To determine continuity, we need to check

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$$

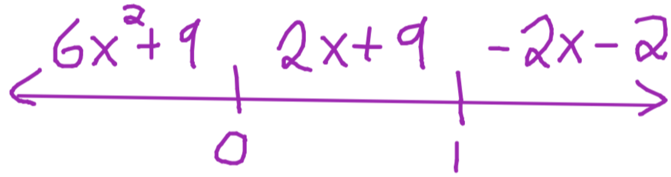
$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$$



$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0}(6x^2+9) = 6(0)^2+9 = 9$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0}(2x+9) = 2(0)+9 = 9$$

No discontinuity
@ $x=0$



$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1}(2x+9) = 2(1)+9 = 11$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1}(-2x-2) = -2(1)-2 = -4$$

Discontinuity @ $x=1$ $\Rightarrow$ Jump $x=1$