Lesson 7: Basic Rules of Differentiation; Derivatives of Sine and Cosine; Derivatives of the Natural Exponential Function

Lesson 7

Basic Rules of Differentiation

① Constant Rule: For any constant c,

$$\frac{d}{dx}[c] = 0$$

Intuitively, this makes sense b/c graph of $f(x)=c$ is a horizontal line.

Proof: Let's take derivative using limit def of derivative.

Let $f(x) = c$.

① $f(x+h) = \cancel{c}$

② $-f(x) = \cancel{-c}$

③ Add ①+② $= 0$

④ Divide by h: $\frac{0}{h} = 0$

⑤ $f'(x) = \lim_{h \to 0} 0 = 0$

Done!

② **Power Rule:** For any real #, n,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Ex 1: Find the derivative of $f(x) = x^2$

$$\frac{d}{dx}(x^2) = 2x^{2-1} = 2x$$

Ex 2: Find the derivative of $f(x) = x^{-4}$

$$\frac{d}{dx}(x^{-4}) = -4x^{-4-1} = -4x^{-5} = -\frac{4}{x^5}$$

③ **Constant Multiple Rule:**

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

④+⑤ **Sum/Difference Rule:**

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

Ex 3: Find the derivative of $f(x) = x^5 + 5x^2$.

$$f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[x^5 + 5x^2]$$

$$= \frac{d}{dx}[x^5] + \frac{d}{dx}[5x^2]$$ (by Rule 4)

$$= \frac{d}{dx}[x^5] + 5\frac{d}{dx}[x^2] \quad \text{(by Rule 3)}$$

$$= 5x^{5-1} + 5 \cdot 2x^{2-1} \quad \text{(by Rule 2)}$$

$$= 5x^4 + 10x$$

Ex 4: Find the derivative of

$$f(x) = \frac{3}{x^4} - 2x^2 + 6x + 7$$

$$\boxed{x^{-m} = \frac{1}{x^m}}$$

We want to rewrite f(x) with no fractions.

$$f(x) = 3x^{-4} - 2x^2 + 6x + 7$$

Now take the derivative like Ex 3.

$$f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[3x^{-4} - 2x^2 + 6x + 7]$$

$$= \frac{d}{dx}[3x^{-4}] - \frac{d}{dx}[2x^2] + \frac{d}{dx}[6x] + \frac{d}{dx}[7] \quad \left(\text{by Rule } 4+5\right)$$

$$= \frac{d}{dx}[3x^{-4}] - \frac{d}{dx}[2x^2] + \frac{d}{dx}[6x] + \frac{d}{dx}[7]$$

$$= 3\frac{d}{dx}[x^{-4}] - 2\frac{d}{dx}[x^2] + 6\frac{d}{dx}[x] + \frac{d}{dx}[7] \quad \left(\text{by Rule } 3\right)$$

$$= 3(-4)x^{-4-1} - 2(2)x^{2-1} + 6x^{1-1} + \frac{d}{dx}[7] \quad \left(\text{by Rule } 2\right)$$

$$= -12x^{-5} - 4x + 6 + \underbrace{\frac{d}{dx}[7]}_{=0} \quad \text{(by Rule 1)}$$

$$= -12x^{-5} - 4x + 6$$

Derivative of Sine and Cosine

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

HW 7.10: Find y' of $y = 3\sin x - 4\cos x$

$$y' = \frac{d}{dx}[3\sin x - 4\cos x]$$

$$= 3\underbrace{\frac{d}{dx}[\sin x]}_{\cos x} - 4\underbrace{\frac{d}{dx}[\cos x]}_{-\sin x}$$

$$= 3\cos x + 4\sin x$$

Derivative of the Natural Exponential Function

$$\frac{d}{dx}(e^x) = e^x$$

HW 7.14: Find the x-value @ which the derivative of $y = 10e^x$ is 1.

i.e. Solve $y' = 1$ for x.

$$y' = \frac{d}{dx}(10e^x) = 10\frac{d}{dx}(e^x) = \underbrace{10e^x = 1}_{\text{Let's solve for } x.}$$

$$10e^x = 1$$

$$e^x = \frac{1}{10}$$

$$\ln(e^x) = \ln\left(\frac{1}{10}\right)$$

$$x = \ln(1/10)$$