Lesson 8: Instantaneous Rates of Change

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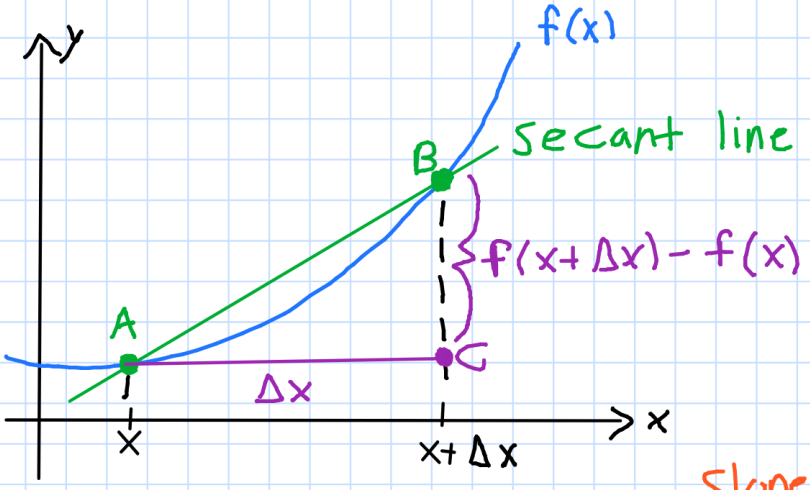

(Recall this image from Lesson 6)

$$\text{slope of secant line} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

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This quantity is also known as the average rate of change.

Average rate of change approaches a quantity is called instantaneous rate of change.
i.e. it's the derivative of f(x)

Ex 1: The initial population of a culture of bacteria is 1000. The population after t hours, $P(t)$, is given by

$$P(t) = 2t^2 + 8t + 1000$$

(a) Find the # of bacteria present after 5 hrs.

i.e. What's $P(5)$?

$P(5) = 2(5)^2 + 8(5) + 1000 = \boxed{1090}$

(b) Find the rate of change of the population after 5 hrs.

i.e. What's $P'(5)$?

$P'(t) = 2(2)t + 8 = 4t + 8$

$P'(5) = 4(5) + 8 = \boxed{28}$

Position & Velocity Functions

Position Function $[s(t)]$ tells us how far away an object is

Velocity Function $[v(t)]$ tells us speed of an object w/ respect to direction

To find Velocity we take the derivative of the position.

$$\boxed{v(t) = s'(t)}$$

Ex 2: An object is shot upward from the surface of Earth. The position function is

$$s(t) = -4.9t^2 + 98t$$

(a) Find $v(t)$.

$v(t) = -4.9(2)t^{2-1} + 98 = -9.8t + 98$

(b) Find $v(3)$.

$v(3) = -9.8(3) + 98 = 68.6$

(c) What is the velocity of the object when it hits the ground?

i.e. Solve $s(t) = 0$ for t. Plug t into $v(t)$.

$0 = s(t) = -4.9t^2 + 98t$

$= -4.9t(t - 20)$

$-4.9t = 0$ | $t - 20 = 0$

$t = 0$ | $t = 20$

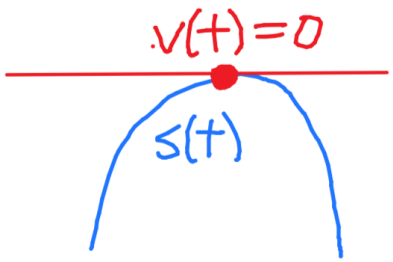
Now plug $t = 20$ for $v(t)$.

Recall $v(t) = -9.8t + 98$

So $v(20) = -9.8(20) + 98 = -98$

(d) When is the object at its highest point?

i.e. Solve $v(t) = 0$ for t.



Recall $v(t) = -9.8t + 98 = 0$

$\frac{98}{9.8} = \frac{9.8t}{9.8}$

$10 = t$

Ex 3: Let $C = 2\pi r$. What is the range of change of C with respect to r?

i.e. Find $\frac{dC}{dr}$.

$$\frac{d}{dr}[C] = \frac{d}{dr}[2\pi r]$$

$$1 \cdot \frac{dC}{dr} = 2\pi \frac{d}{dr}[r]$$

$$\frac{dC}{dr} = 2\pi \cdot 1 \frac{dr}{dr}^{1} = 2\pi$$

Hence $\frac{dC}{dr} = 2\pi$

Ex 4: Let $p = 3q - 5$

@ What is the rate of change of p with respect to q?

i.e. Find $\frac{dp}{dq}$

$$\frac{d}{dq}[p] = \frac{d}{dq}[3q - 5]$$

$$1 \cdot \frac{dp}{dq} = 3\frac{d}{dq}[q] - \frac{d}{dq}[5]$$

$$\frac{dp}{dq} = 3\frac{dq}{dq}^{1} - 0$$

$$\frac{dp}{dq} = 3$$

Ex 4: Let $p = 3q - 5$

(b) What is the rate of change of q with respect to p?

i.e. Find $\frac{dq}{dp}$

$$\frac{d}{dp}[p] = \frac{d}{dp}[3q - 5]$$

$$1 \cdot \frac{dp}{dp} = 3\frac{d}{dp}[q] - \frac{d}{dp}[5]$$

$$\frac{dp}{dp}^{1} = 3\frac{dq}{dp} - 0$$

$$1 = 3\frac{dq}{dp}$$

$$\frac{1}{3} = \frac{dq}{dp}$$