Lesson 9: Product Rule

Lesson 9: Product Rule

Product Rule says the derivative of $h(x)=u(x)v(x)$ is

$$\frac{d}{dx}[h(x)]=\frac{d}{dx}[u(x)v(x)]$$
$$=\frac{d}{dx}[u(x)]v(x)+u(x)\frac{d}{dx}[v(x)]$$
$$=u'(x)v(x)+u(x)v'(x)$$

Ex 1: Given $h(x)=2x^3e^x$. Compute $h'(x)$

i.e. Find $h'(x)$ by product rule.

Let $u(x)=2x^3$ $\quad v(x)=e^x$

$u'(x)=6x^2$ $\quad v'(x)=e^x$

By product rule,

$$h'(x)=u'(x)v(x)+u(x)v'(x)$$
$$=\boxed{6x^2e^x+2x^3e^x}$$

Ex 2: Given $h(x)=x^2\sin x$. Compute $h'(\pi/6)$

i.e. Find $h'(x)$ by product rule. Then plug $\frac{\pi}{6}$ for x.

Let $u(x)=x^2$ $\quad v(x)=\sin x$

$u'(x)=2x$ $\quad v'(x)=\cos x$

By product rule,

$$h'(x)=u'(x)v(x)+u(x)v'(x)$$
$$=2x\sin x+x^2\cos x$$

$$h'\left(\frac{\pi}{6}\right) = \frac{2\pi}{6}\sin\left(\frac{\pi}{6}\right) + \left(\frac{\pi}{6}\right)^2\cos\left(\frac{\pi}{6}\right)$$
$$= \frac{\pi}{3}\cdot\frac{1}{2} + \frac{\pi^2}{36}\cdot\frac{\sqrt{3}}{2}$$
$$= \boxed{\frac{\pi}{6} + \frac{\sqrt{3}\pi^2}{72}}$$

Ex3: Given $h(x) = \sqrt{x}(2x^2+4)$. Compute $h'(x)$

Method 1: Find $h'(x)$ by product rule.

Let $u(x) = \sqrt{x} = x^{1/2}$ $\qquad$ $v(x) = 2x^2+4$

$u'(x) = \frac{1}{2}x^{-1/2}$ $\qquad$ $v'(x) = 4x$

By product rule,
$$h'(x) = u'(x)v(x) + u(x)v'(x)$$
$$= \frac{1}{2}x^{-1/2}(2x^2+4) + x^{1/2}(4x)$$
$$= x^{3/2} + 2x^{-1/2} + 4x^{3/2} = \boxed{5x^{3/2} + 2x^{-1/2}}$$

Method 2: Expand $h(x)$ to use Power Rule

$$h(x) = x^{1/2}(2x^2+4)$$
$$= 2x^{5/2} + 4x^{1/2}$$

By power rule,
$$h'(x) = 2\cdot\frac{5}{2}x^{3/2} + 4\cdot\frac{1}{2}x^{-1/2}$$
$$= \boxed{5x^{3/2} + 2x^{-1/2}}$$

Moral: Just b/c there is a product doesn't mean you need to use product rule

HW 9.2: Given $h(x)=(x^2+5x)(-3x^5+6)$. Find $h'(x)$

Let's expand $h(x)$.

	$-3x^5$	6
x^2	$-3x^7$	$6x^2$
$5x$	$-15x^6$	$30x$

(Note this method works for m-terms by n-terms)

So $h(x)=-3x^7-15x^6+6x^2+30x$

Hence $h'(x)=\boxed{-21x^6-90x^5+12x+30}$

HW 9.4: Find x-values @ which $y=4x^6e^x$ has a horizontal tangent.

i.e. Solve $y'(x)=0$ for x.

To find y' use product rule. Let

$u(x)=4x^6 \qquad v(x)=e^x$

$u'(x)=24x^5 \qquad v'(x)=e^x$

By product rule,

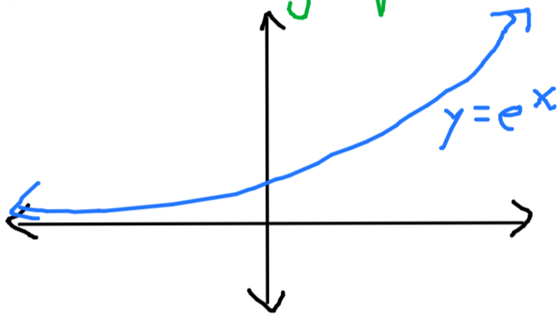
$$y'(x)=u'(x)v(x)+u(x)v'(x)$$
$$=24x^5e^x+4x^6e^x=0$$

To solve $y'(x)\overset{\cup}{=}0$ for x we need to factor.

$$y'(x)=24x^5e^x+4x^6e^x=0$$
$$4x^5e^x(6+x)=0$$

$4x^5=0$	$e^x=0$	$6+x=0$
$x=0$	$\downarrow$ Never happens	$x=-6$

Recall the graph of e^x.



Does the graph cross x-axis? No

Hence the final answers are $x=0$, $x=-6$