MA 16010: Applied Calculus 1 Quiz 11 (Solutions) Fri. Nov. 5, 2021

Please show **all** your work! Answers without supporting work will not be given cblueit.
Write answers in spaces provided.

Name:____

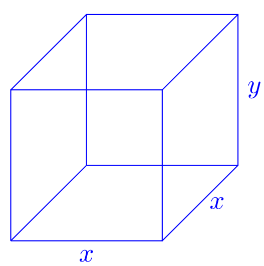
1. **[1 pt]** **[10 pts]** A box with a square base and open top is to be made from 100 square inches of material. What is the volume of the largest box that can be made?

To receive full credit for this problem, you must show all 7 steps, as discussed in lecture.

Solution: **Steps:**

(1) **[1 pt]** Volume, V

(2) **[1 pt]** Open-Top Box



(3) **[1 pt]** $V = x^2 y$

(4) **[1 pt]** $100 = A = x^2 + 4xy$

(5) **[1 pt]** Domain of x: $(0, 10)$
Domain of y: $(0, \infty)$

(6) **[3 pts]** Solve (4) for y.

$$100 = x^2 + 4xy$$
$$100 - x^2 = 4xy$$
$$\frac{100 - x^2}{4x} = y$$

Plug y into (3).

$$\begin{aligned} V &= x^2 y \\ &= x^2\left(\frac{100 - x^2}{4x}\right) \\ &= \frac{x}{4}(100 - x^2) \\ &= 25x - \frac{x^3}{4} \end{aligned}$$

Take the derivative and set $= 0$.

$$V' = 25 - \frac{3}{4}x^2 = 0$$
$$25 = \frac{3}{4}x^2$$
$$\frac{100}{3} = x^2$$
$$x = \sqrt{\frac{100}{3}}$$

Now we need to check for absolute maximum.
By the Second Derivative Test,

$$V'' = -\frac{3}{4}(2)x = -\frac{3}{2}x$$
$$V''\left(\sqrt{\frac{100}{3}}\right) = -\frac{3}{2} \cdot \sqrt{\frac{100}{3}} < 0$$

Hence $x = \sqrt{\frac{100}{3}}$ is an absolute maximum.

(7) **[1 pt]** Answer: $x = \sqrt{\dfrac{100}{3}}$. So

$$y = \frac{100 - x^2}{4x} = \frac{100 - \dfrac{100}{3}}{4 \cdot \sqrt{\dfrac{100}{3}}} = \frac{5\sqrt{3}}{3}$$

Hence the volume of such box is

$$V = \frac{500\sqrt{3}}{9}$$