MA 16010: Applied Calculus 1 Quiz 6 (Solutions) Fri. Oct 1, 2021

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:_____

1. **[5pts]** Find the derivative of $y = \ln\sqrt{\dfrac{4x+3}{x^2+2}}$

Solution: Method 1: Rewrite y:

$$\begin{aligned} y &= \ln\sqrt{\frac{4x+3}{x^2+2}} \\ &= \ln\left(\frac{4x+3}{x^2+2}\right)^{1/2} \\ &= \frac{1}{2}\ln\frac{4x+3}{x^2+2} \\ &= \frac{1}{2}\left[\ln(4x+3) - \ln(x^2+2)\right] \\ &= \frac{1}{2}\ln(4x+3) - \frac{1}{2}\ln(x^2+2) \quad \textbf{[3pts]} \end{aligned}$$

Now differentiate. Note each of these logarithms are composition of functions, so use Chain Rule when differentiating.

$$y' = \frac{1}{2}\cdot\frac{1}{4x+3}\cdot(4) - \frac{1}{2}\cdot\frac{1}{x^2+2}\cdot(2x) = \boxed{\frac{2}{4x+3} - \frac{x}{x^2+2}} \quad \textbf{[2pts]}$$

Method 2: Find the derivative using Chain Rule, then Chain Rule, then Quotient Rule. Hence

$$y' = \underbrace{\frac{1}{\left(\dfrac{4x+3}{x^2+2}\right)^{1/2}}}_{2\text{ pts}} \cdot \underbrace{\frac{1}{2}\left(\frac{4x+3}{x^2+2}\right)^{-1/2}}_{2\text{ pts}} \cdot \underbrace{\frac{4(x^2+2)-(4x+3)(2x)}{(x^2+2)^2}}_{1\text{ pt}}$$

2. **[5pts]** Find the second derivative of $y = \sin^2(x)$. **Simplify!**

Solution: Rewrite y:

$$y = \sin^2(x) = (\sin x)^2$$

To find the second derivative, we need to find the first derivative first. By Chain Rule,

$$y' = 2\sin x\cdot\cos x \quad \textbf{[2pt]}$$

To find the second derivative, take the derivative of y'. Note to do so, you need to use Product Rule.

$$y'' = 2\sin x \cdot \sin x + 2(-\cos x) \cdot \cos x \quad \textbf{[2pt]}$$

$$= \boxed{2\sin^2 x - 2\cos^2 x} \quad \textbf{[1pt]}$$