MA 16010: Applied Calculus 1 Quiz 8 (Solutions) Fri. Oct. 22, 2021

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:______________________________

1. **[5pts]** Consider the function $f(x) = \frac{x^5}{10} + \frac{x^4}{4} + 2$. Find the x-coordinate of all the inflection points.

Solution: First, we need to find the x-values for which $f''(x) = 0$.

$$f'(x) = \frac{x^4}{2} + x^3 \qquad \textbf{[1 pt]}$$
$$f''(x) = 2x^3 + 3x^2 = x^2(2x+3) = 0 \qquad \textbf{[1 pt]}$$

Hence $x = 0, -3/2$. **[1 pt]**

Recall that an inflection point is a point where the concavity change. To determine so, we use a sign chart (i.e. number line) with f''.

$$\begin{array}{ccccc} - & & + & & + \\ -2 & & -1 & & 1 \\ \longleftarrow & -\frac{3}{2} & \longrightarrow & 0 & \longrightarrow \end{array} \quad f'' \qquad \textbf{[1 pt]}$$

From the sign chart above, we see there is only one sign change, at $x = 0$. i.e. The x-coordinate of the inflection point is 0. **[1 pt]**

2. **[5pts]** Consider the function $f(x) = 2\ln(x^2+1)$. Find the largest open interval(s) on which the function is both concave up and increasing.

Solution: Remember to determine increasing/decreasing we need to create a sign chart (i.e. number line) using f'. So,

$$f'(x) = 2 \cdot \frac{1}{x^2+1} \cdot 2x = \frac{4x}{x^2+1} = 0 \qquad \textbf{[1pt]}$$

Hence $f'(x) = 0$ when $4x = 0$. Hence $x = 0$ is a critical point. Now, we create a sign chart (i.e. number line) with f'.

$$\begin{array}{ccc} - & & + \\ -1 & & 1 \\ \longleftarrow & 0 & \longrightarrow \end{array} \quad f' \qquad \textbf{[1 pt]}$$

Remember to determine concavity we need to create a sign chart (i.e. number line) using f''. So,

$$\begin{aligned} f''(x) &= \frac{4(x^2+1) - 4x(2x)}{(x^2+1)^2} \\ &= \frac{4x^2 + 4 - 8x^2}{(x^2+1)^2} \\ &= \frac{4 - 4x^2}{(x^2+1)^2} \\ &= \frac{4(1-x^2)}{(x^2+1)^2} \\ &= \frac{4(1-x)(1+x)}{(x^2+1)^2} = 0 \qquad \textbf{[1pt]} \end{aligned}$$

Hence $f''(x) = 0$ when $x = \pm 1$. Now, we create a sign chart (i.e. number line) with f''.

$$\begin{array}{ccccc} - & & + & & - \\ -2 & & 0 & & 2 \\ \longleftarrow & -1 & \text{———} & 1 & \longrightarrow \end{array} \quad f' \quad \textbf{[1 pt]}$$

Combining both number lines, we have the following information:

	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
f'	$-$	$-$	$+$	$+$
f''	$-$	$+$	$+$	$-$

[1 pt]

We want concave up ($+$ for f'') and increasing ($+$ for f'), which only occurs at $(0, 1)$. **[1 pt]**